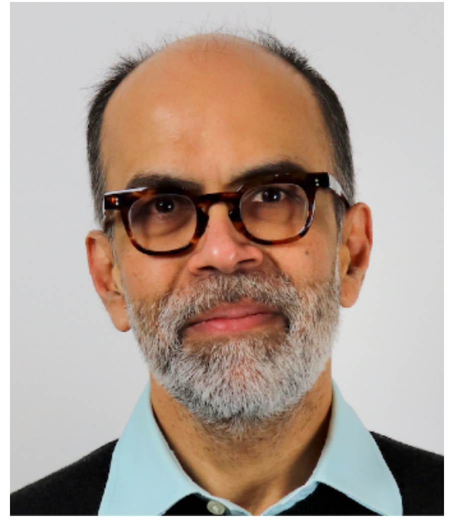
Pairwise-Independent Contention Resolution



Anupam Gupta
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July 3, 2024

IPCO 2024

Matroid Prophet Inequalities

[Kleinberg Weinberg ’12]

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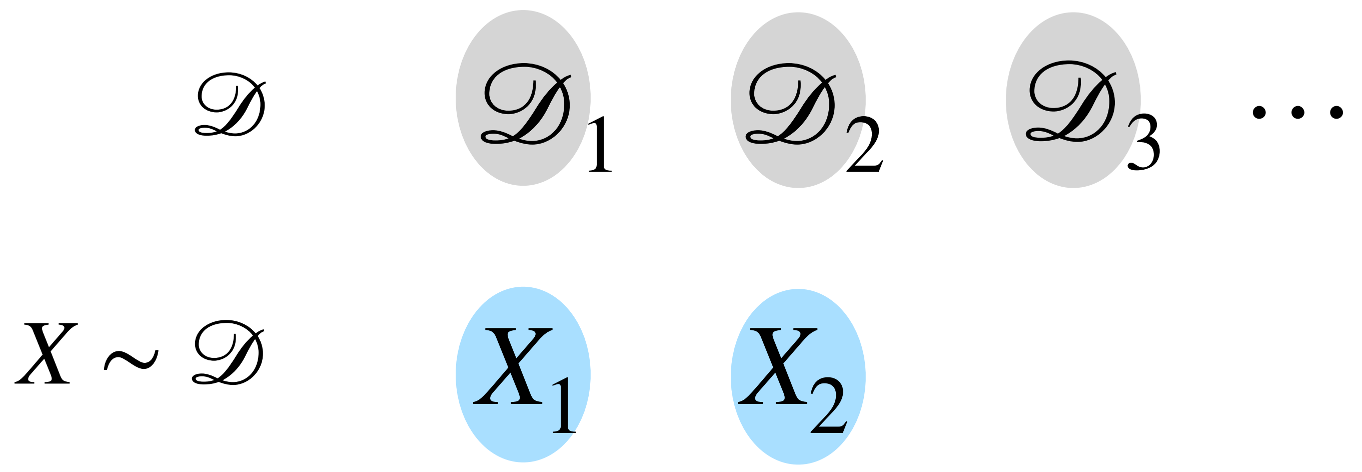

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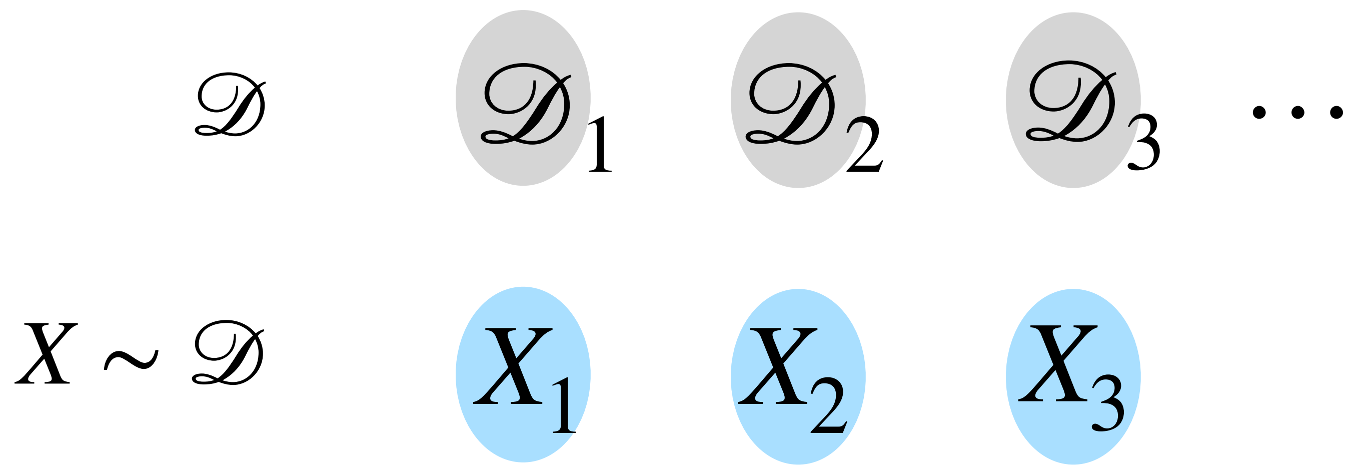

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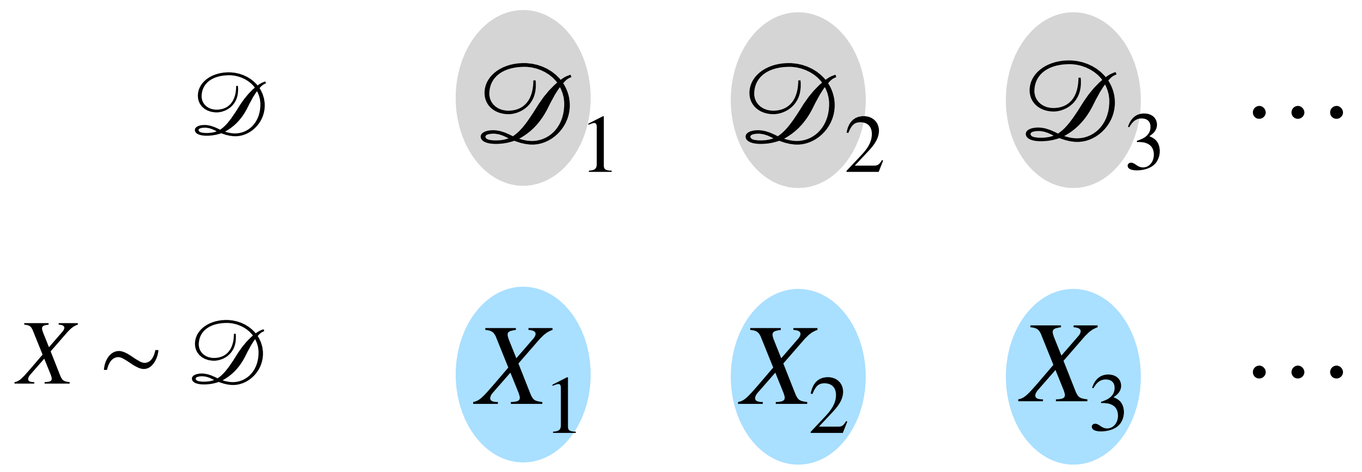

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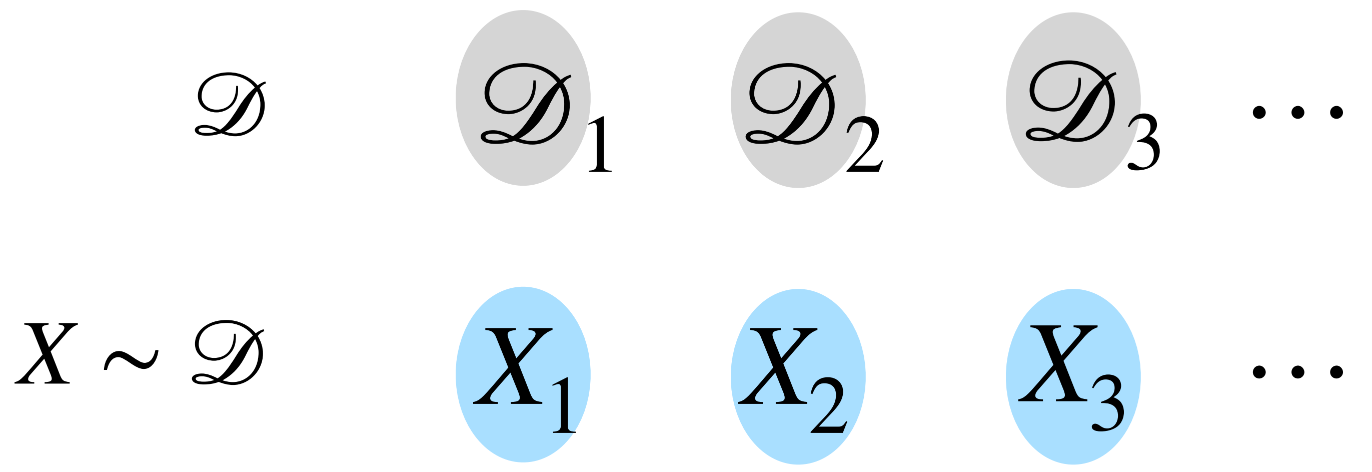

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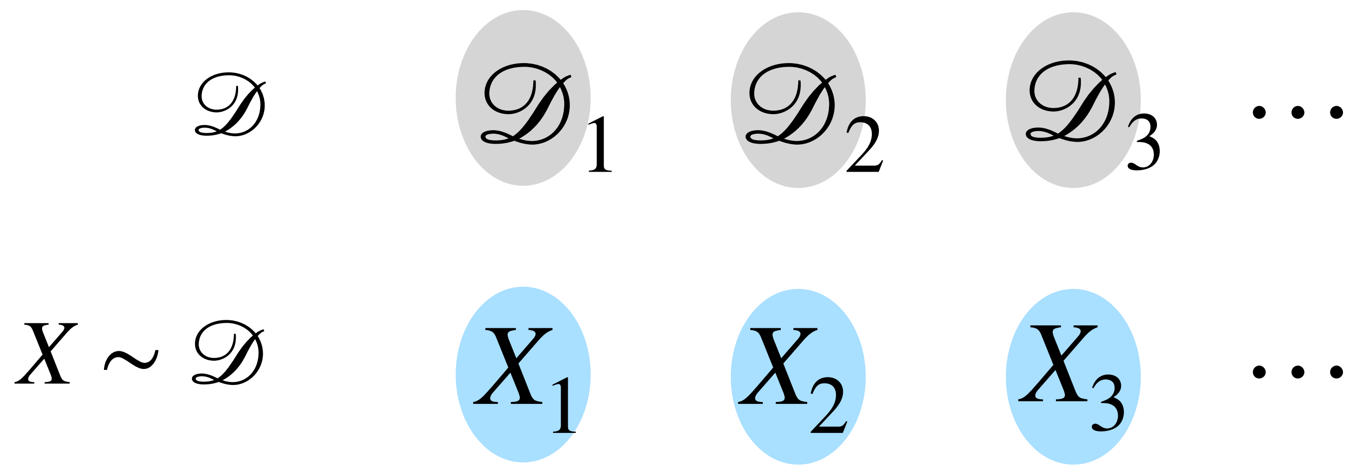
Theorem [Kleinberg Weinberg 12]:

There is an $\Omega(1)$-competitive policy when $\mathscr{F}$ is a matroid.



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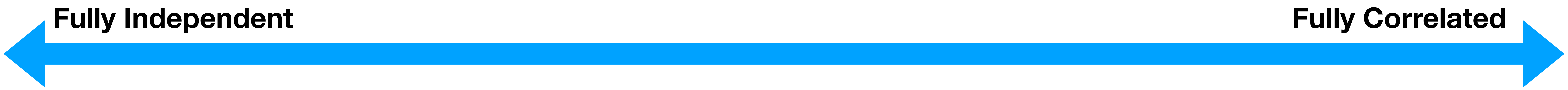

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OCRSes with Pairwise Independence
(matroid constraint)
[this work]

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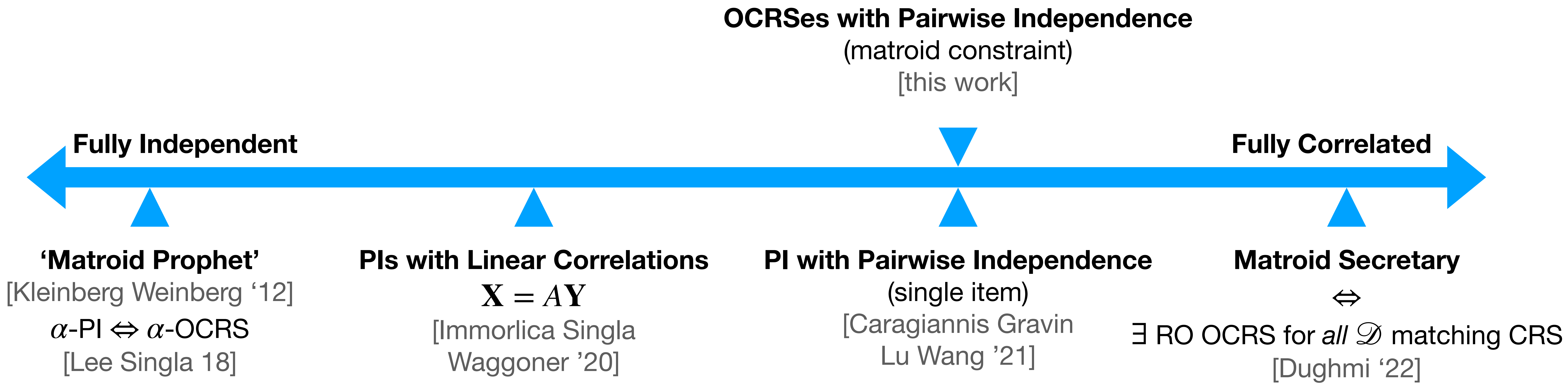

... Which classes of matroids admit good pairwise-independent prophet inequalities, OCRSes?

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[Feldman Svensson Zenklusen '16]

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Note:

1. OCRSs are *greedy*, meaning: alg. chooses downward-closed $\mathscr{F} \subseteq \mathscr{M}$, accepts arriving i iff $I \cup \{i\} \in \mathscr{F}$.

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[Feldman Svensson Zenklusen '16]

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What is the joint distribution $\mathscr{D}$?

$\mathscr{D}$ is *pairwise-consistent* with x if:

- $\Pr[R_i = 1] = x_i$ for all i
- $\Pr[R_i = 1 \ \wedge \ R_j = 1] = x_i x_j$ for all i, j

The Single-Item Setting

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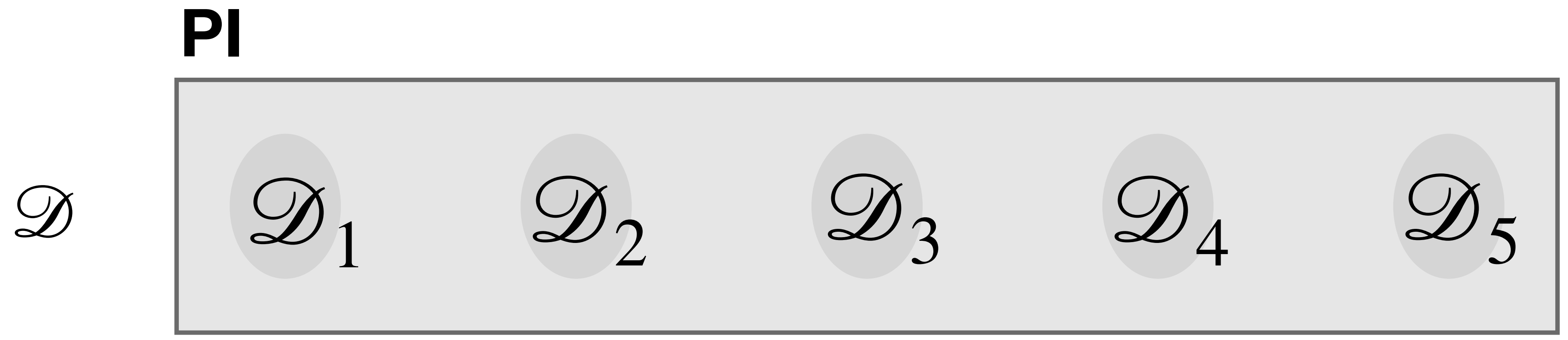

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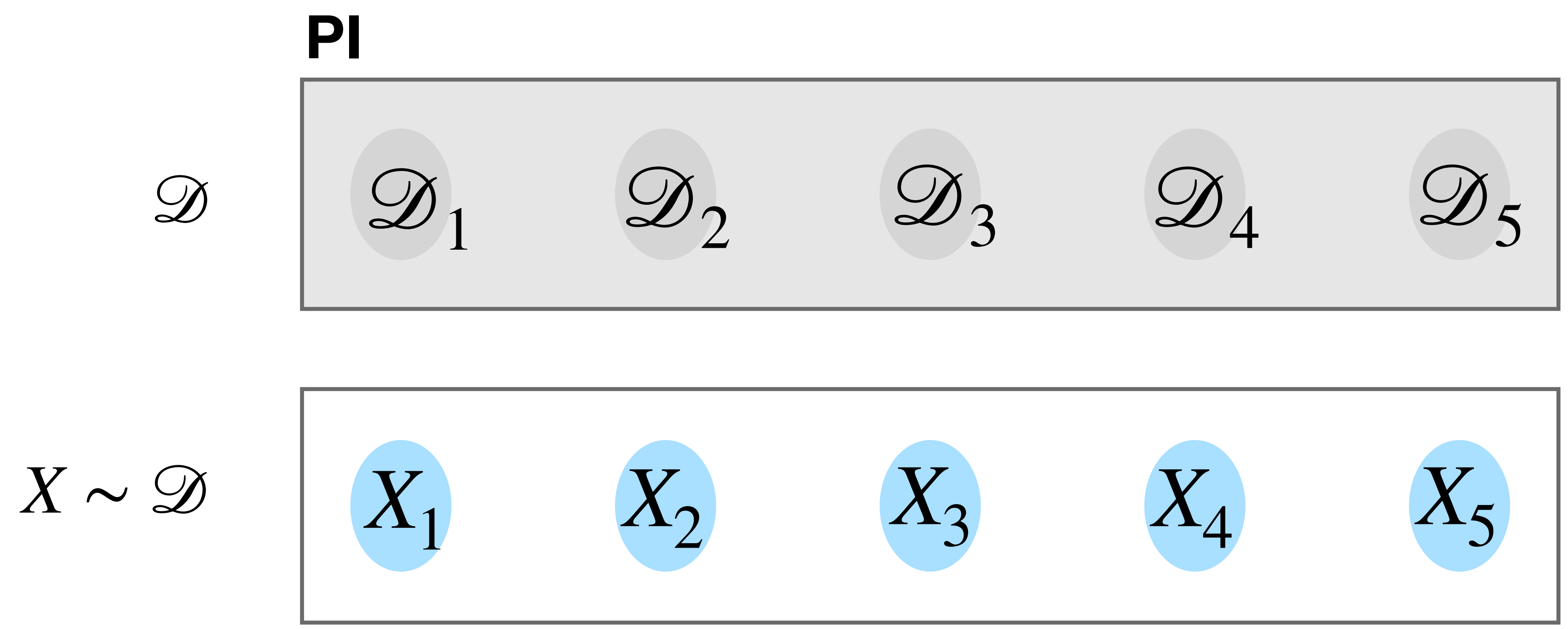

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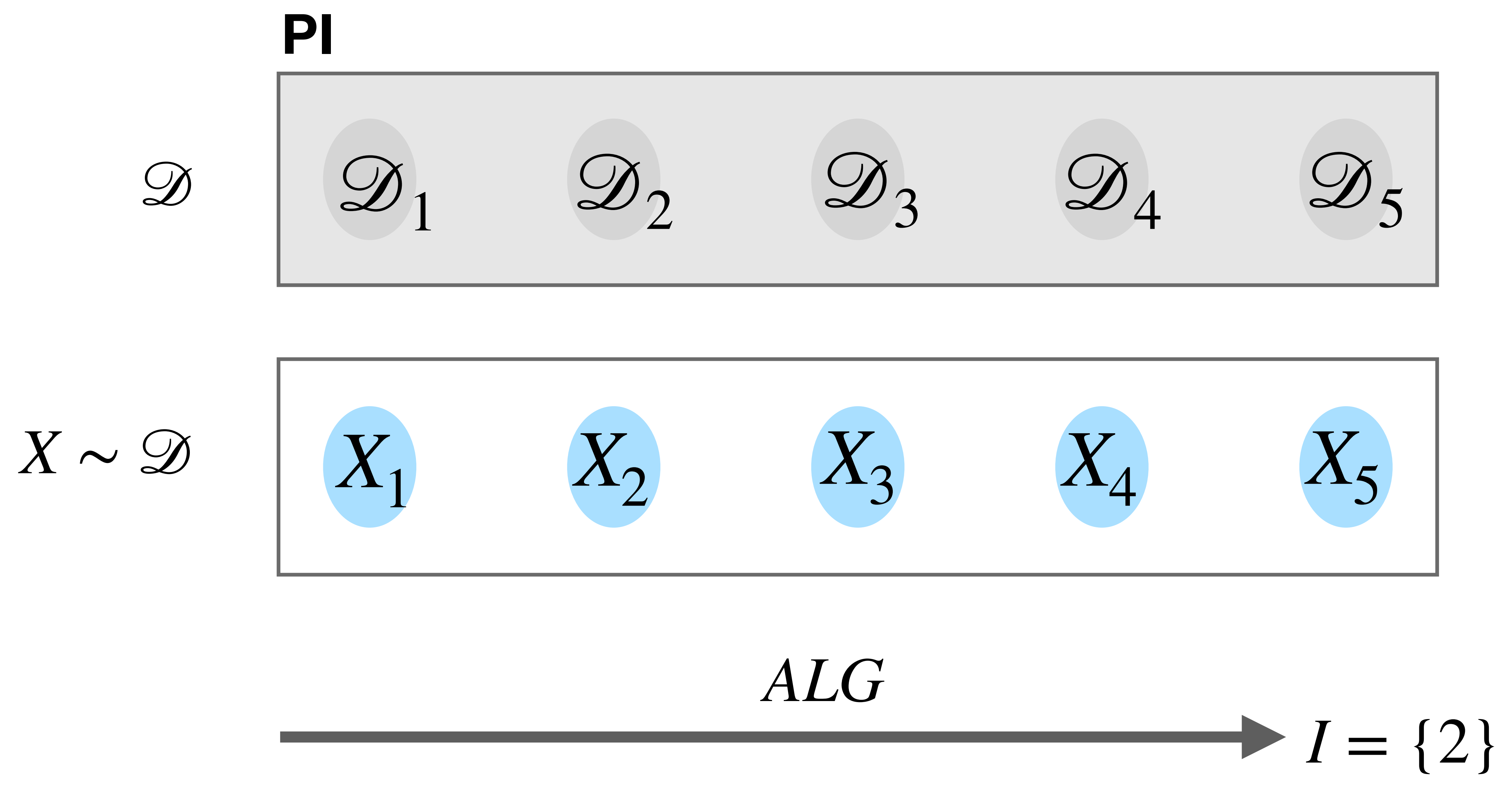

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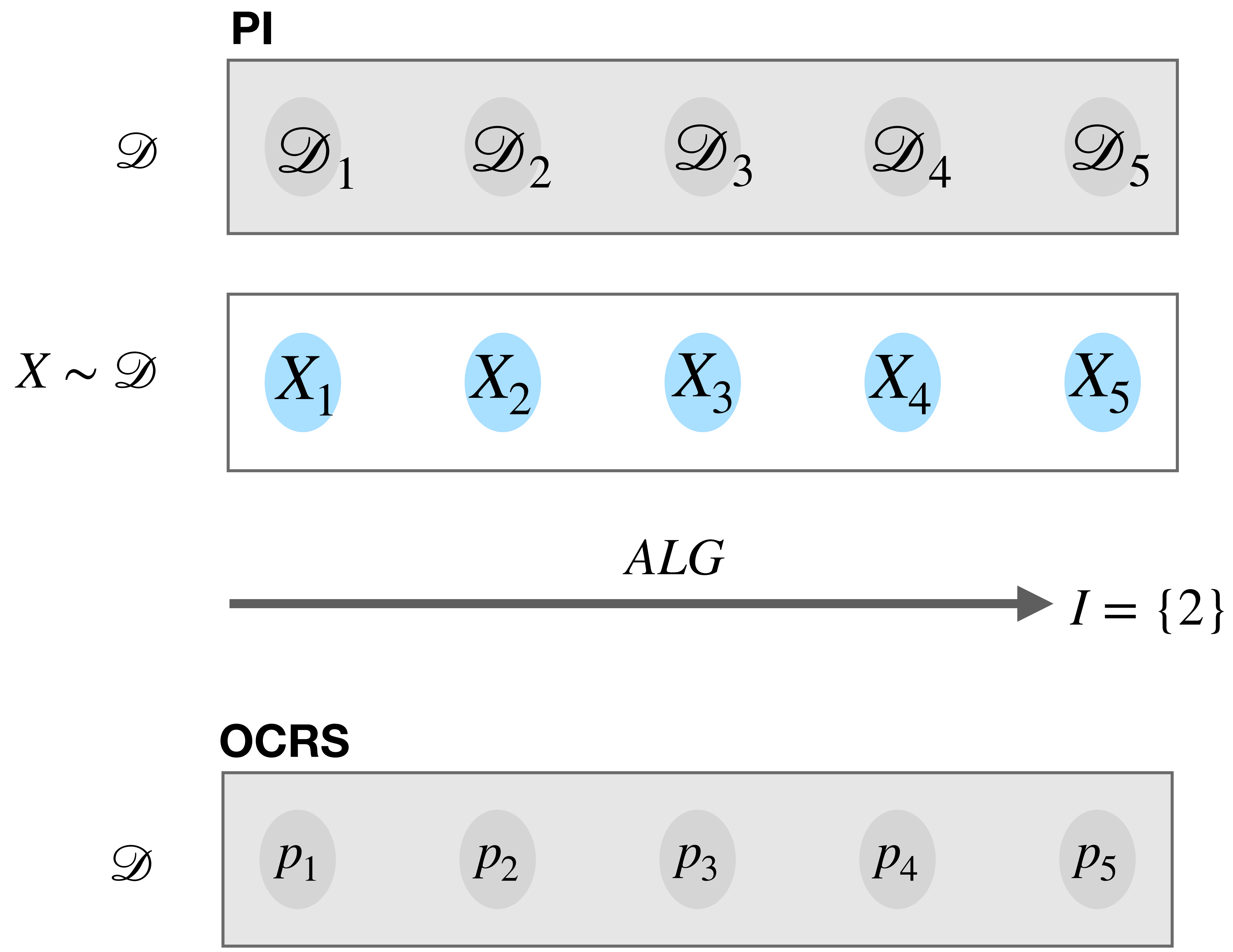

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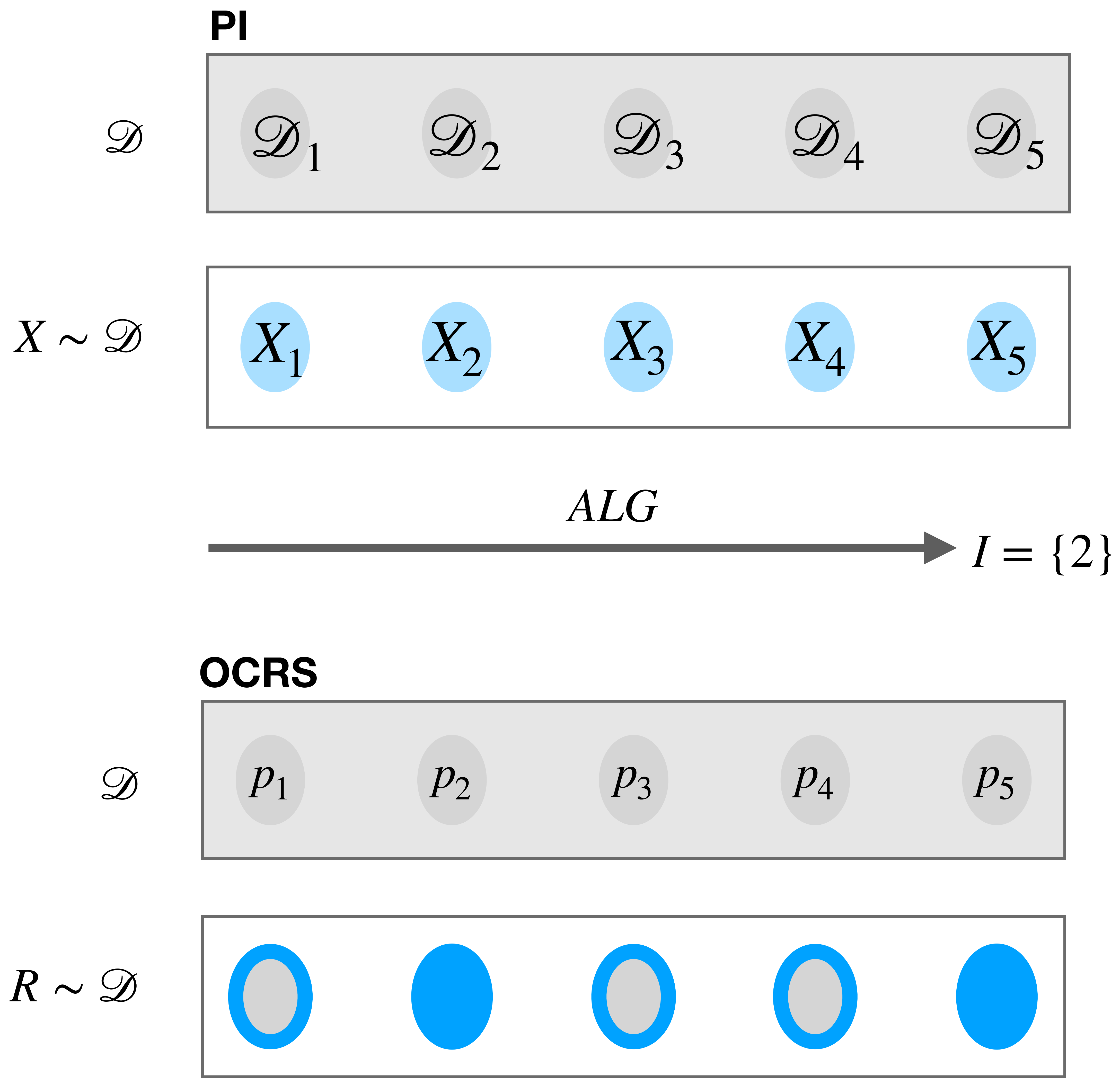

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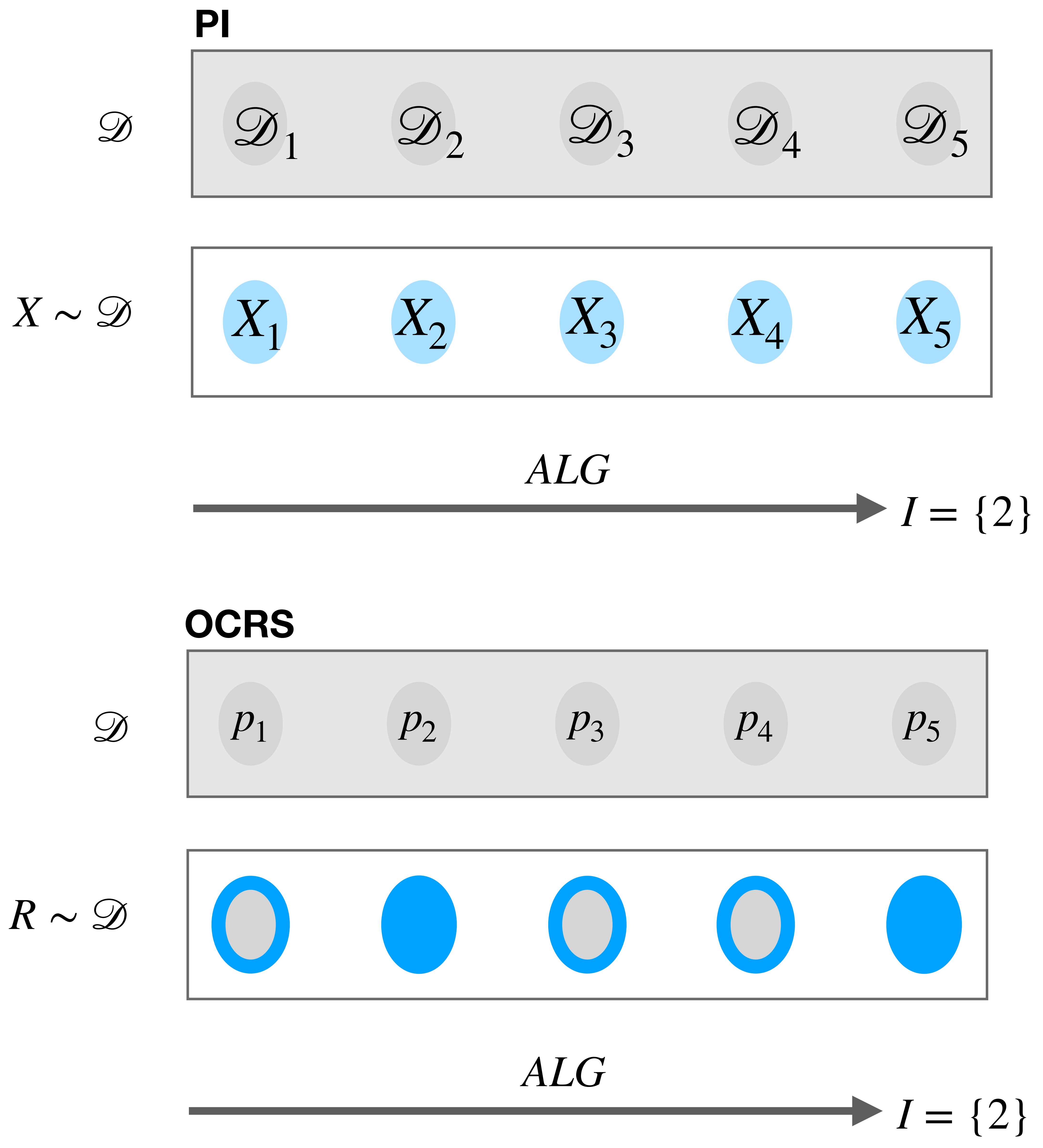

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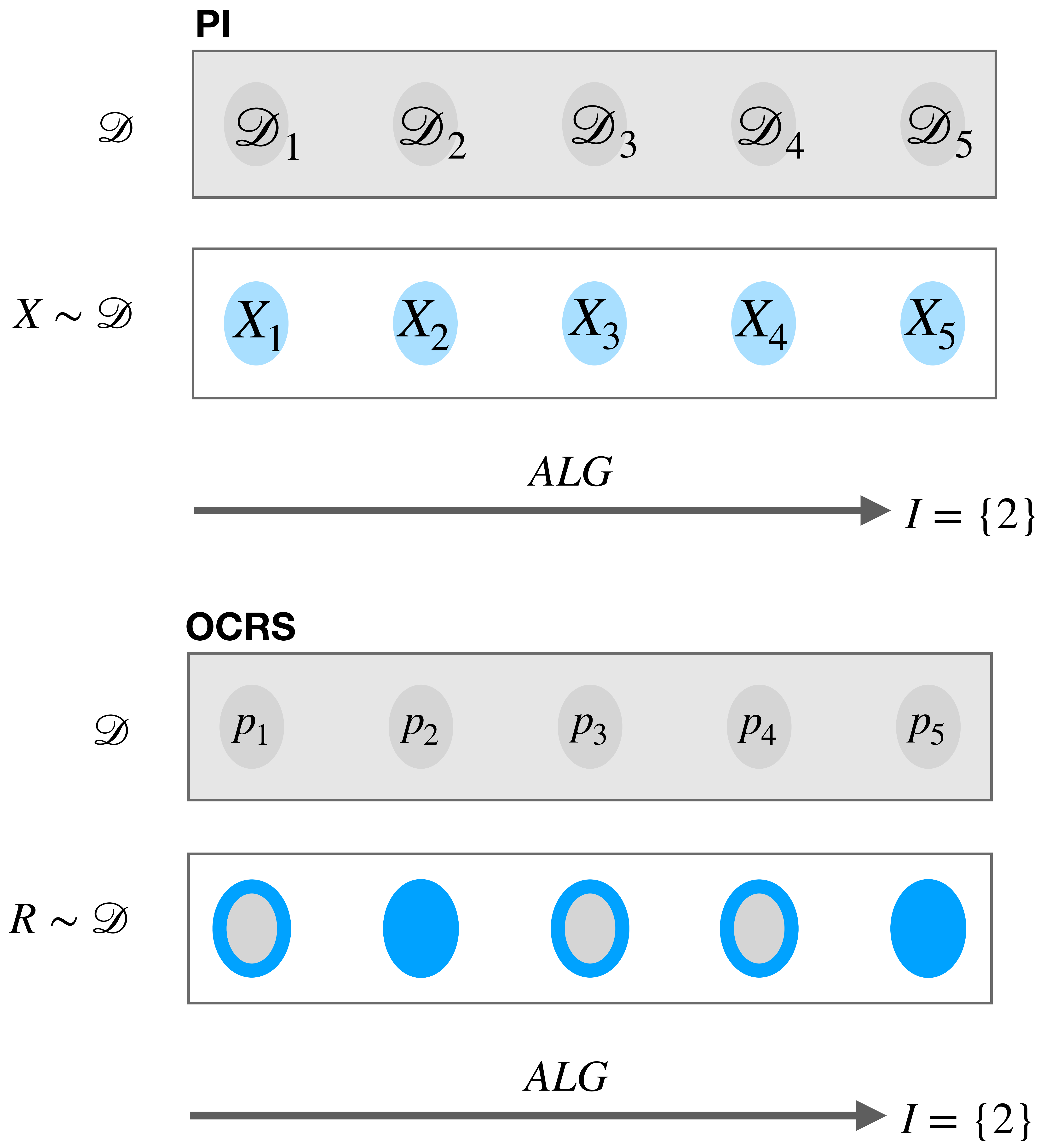

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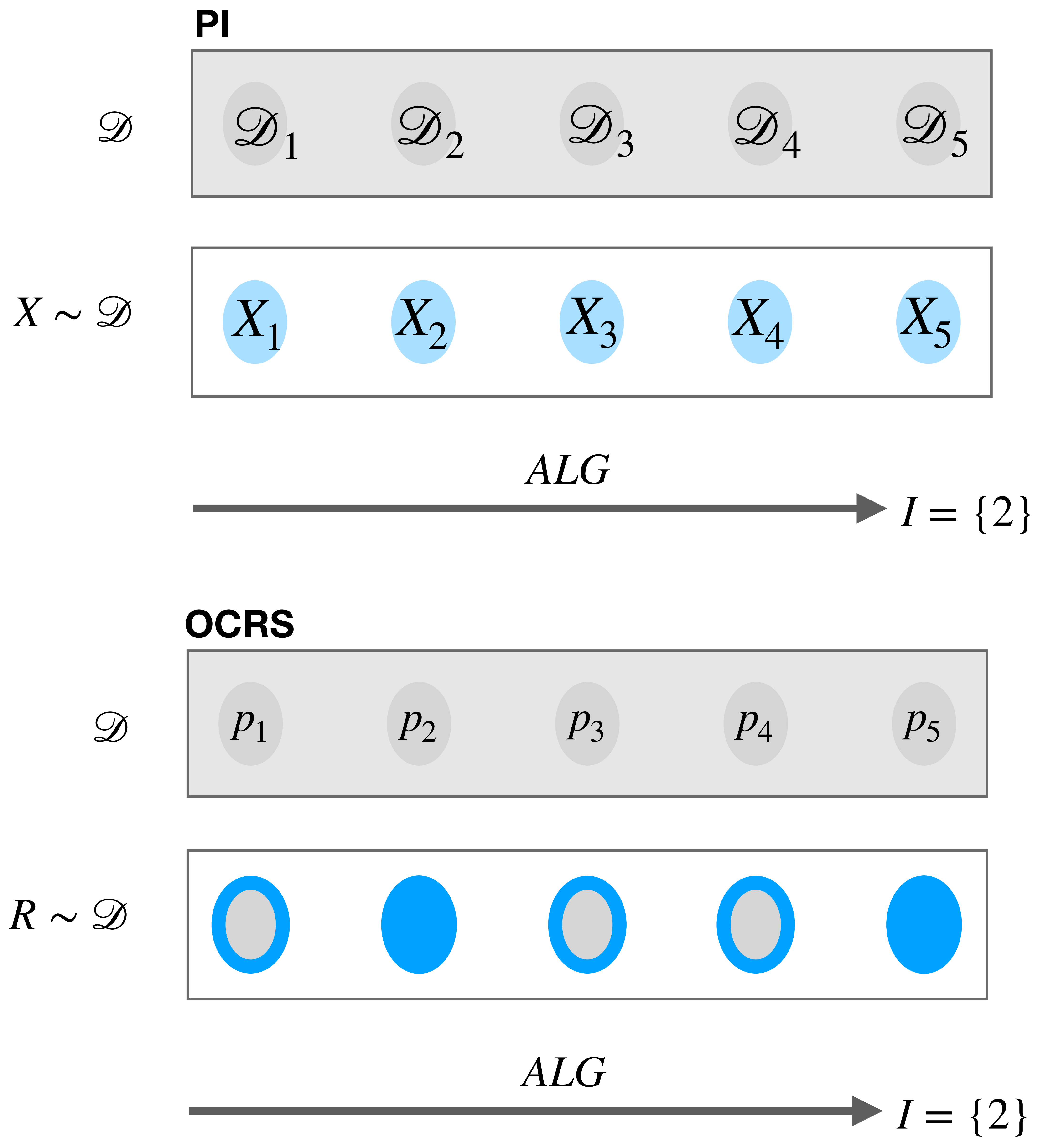

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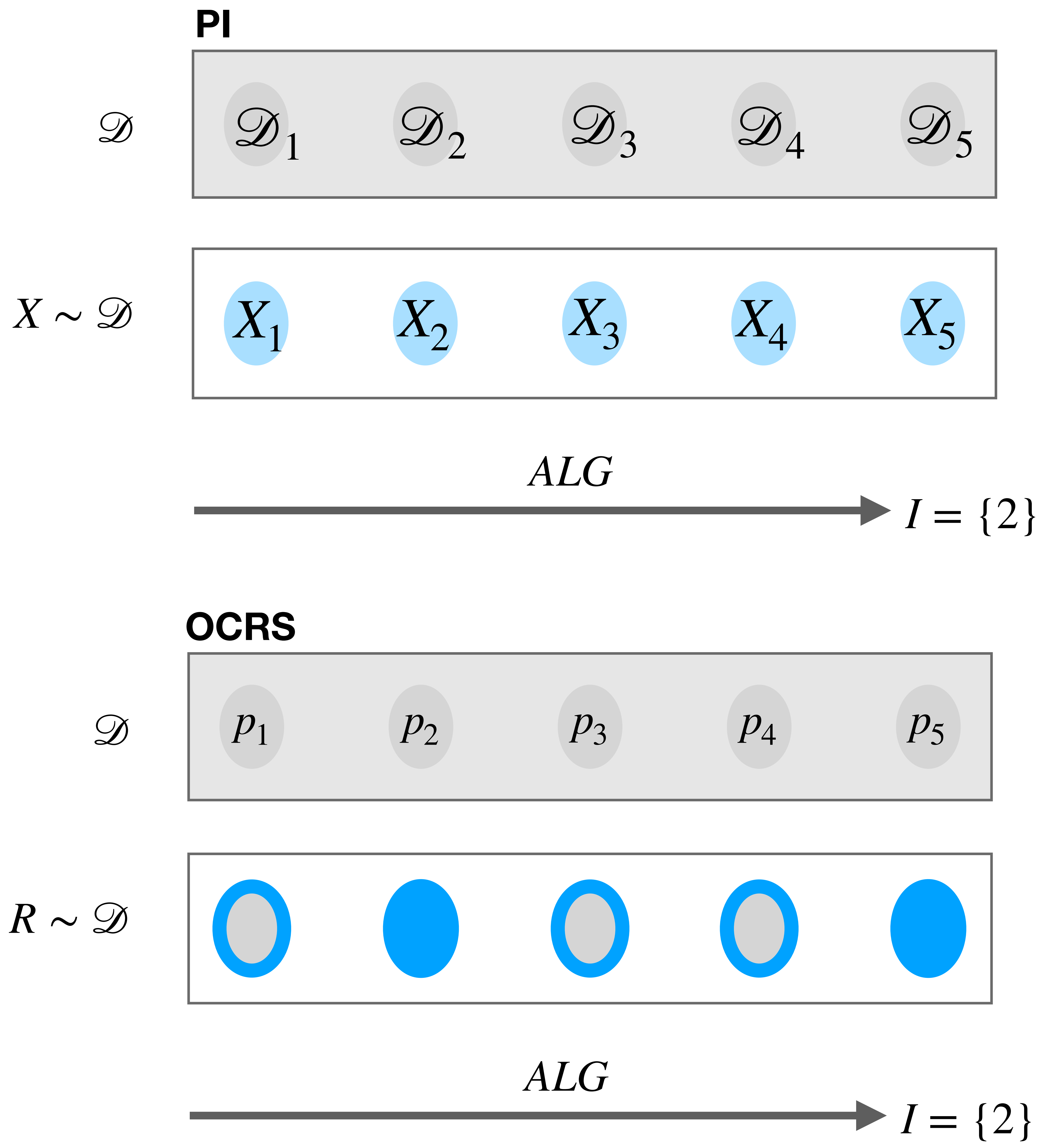

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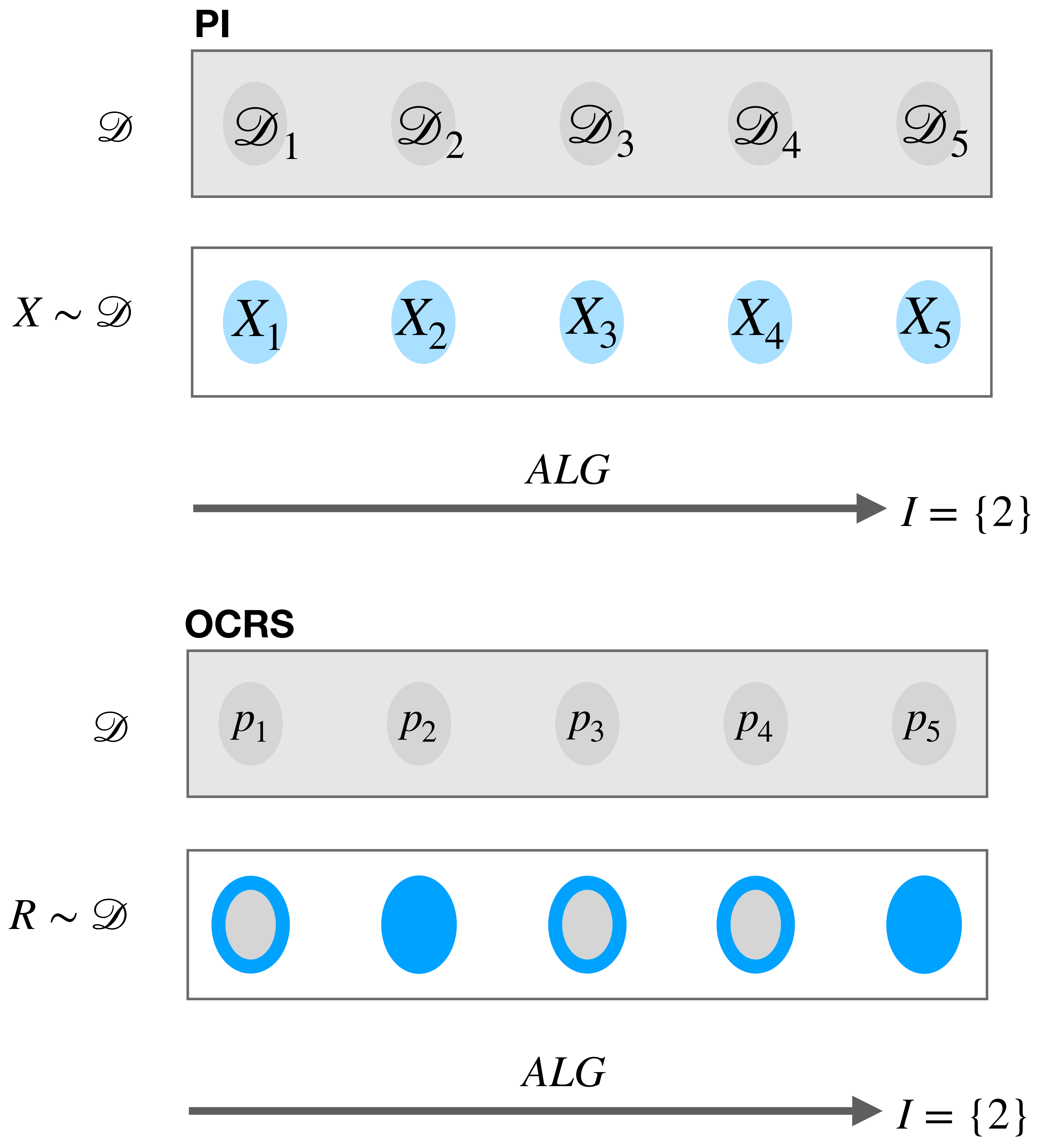

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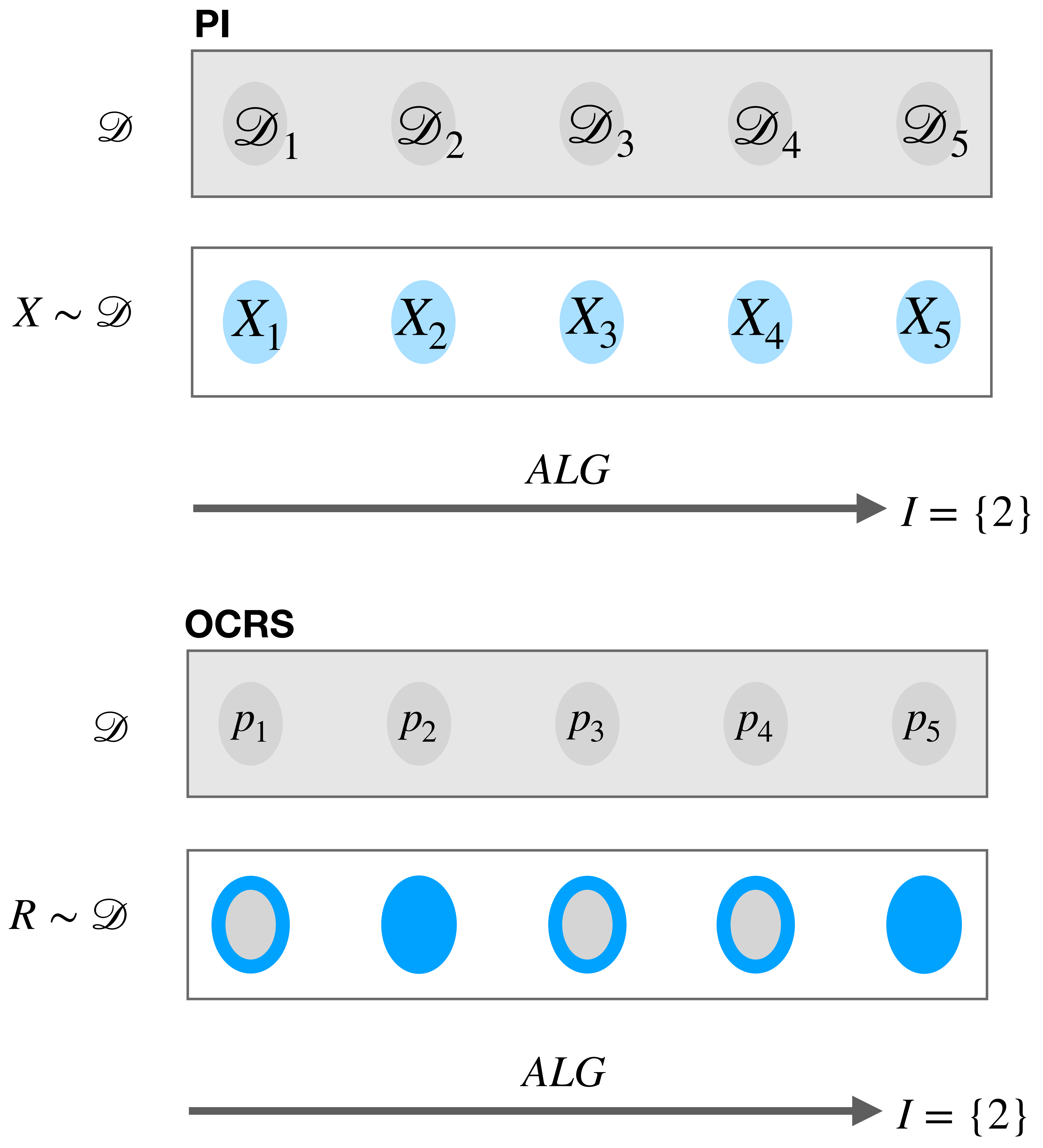

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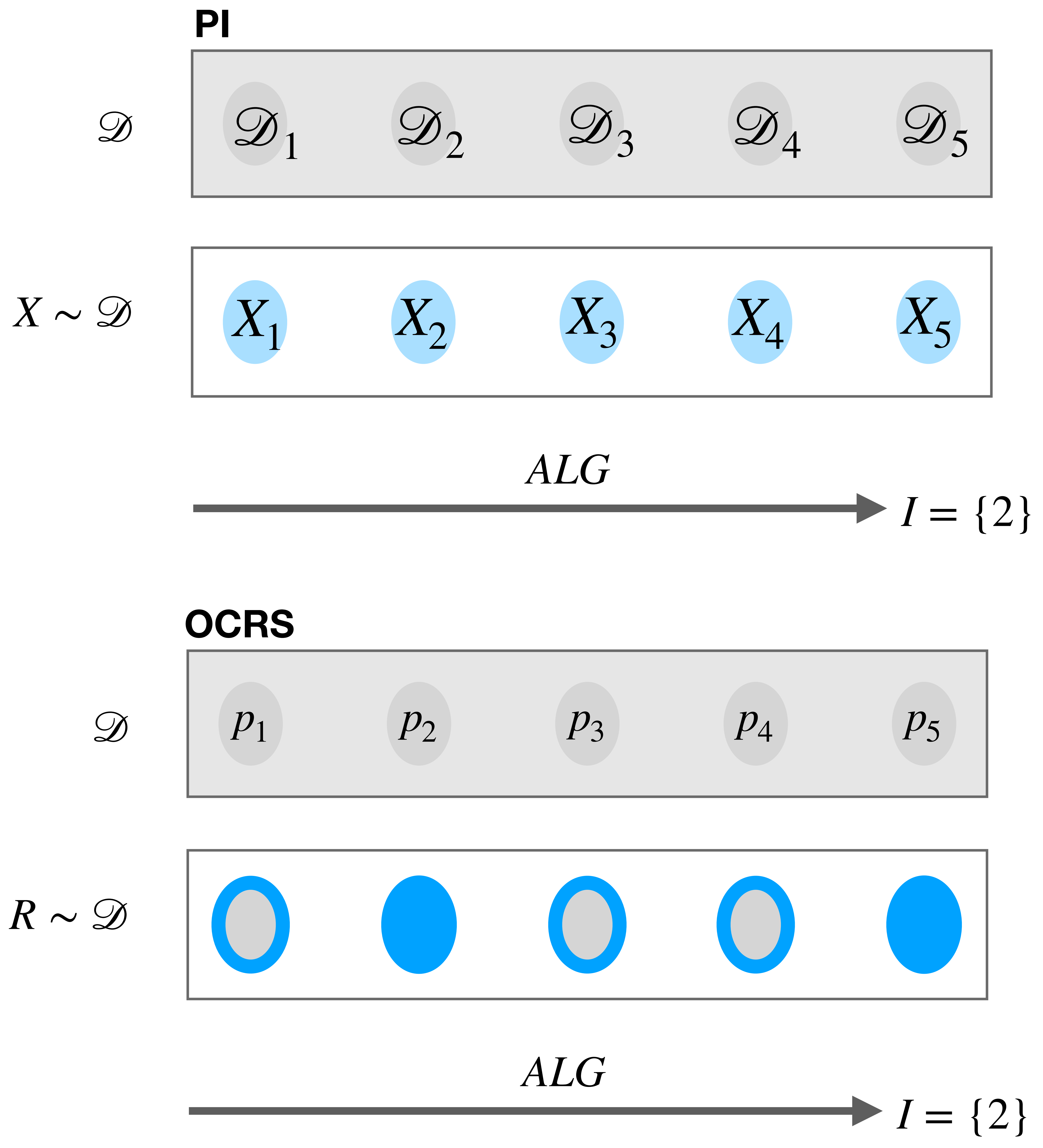

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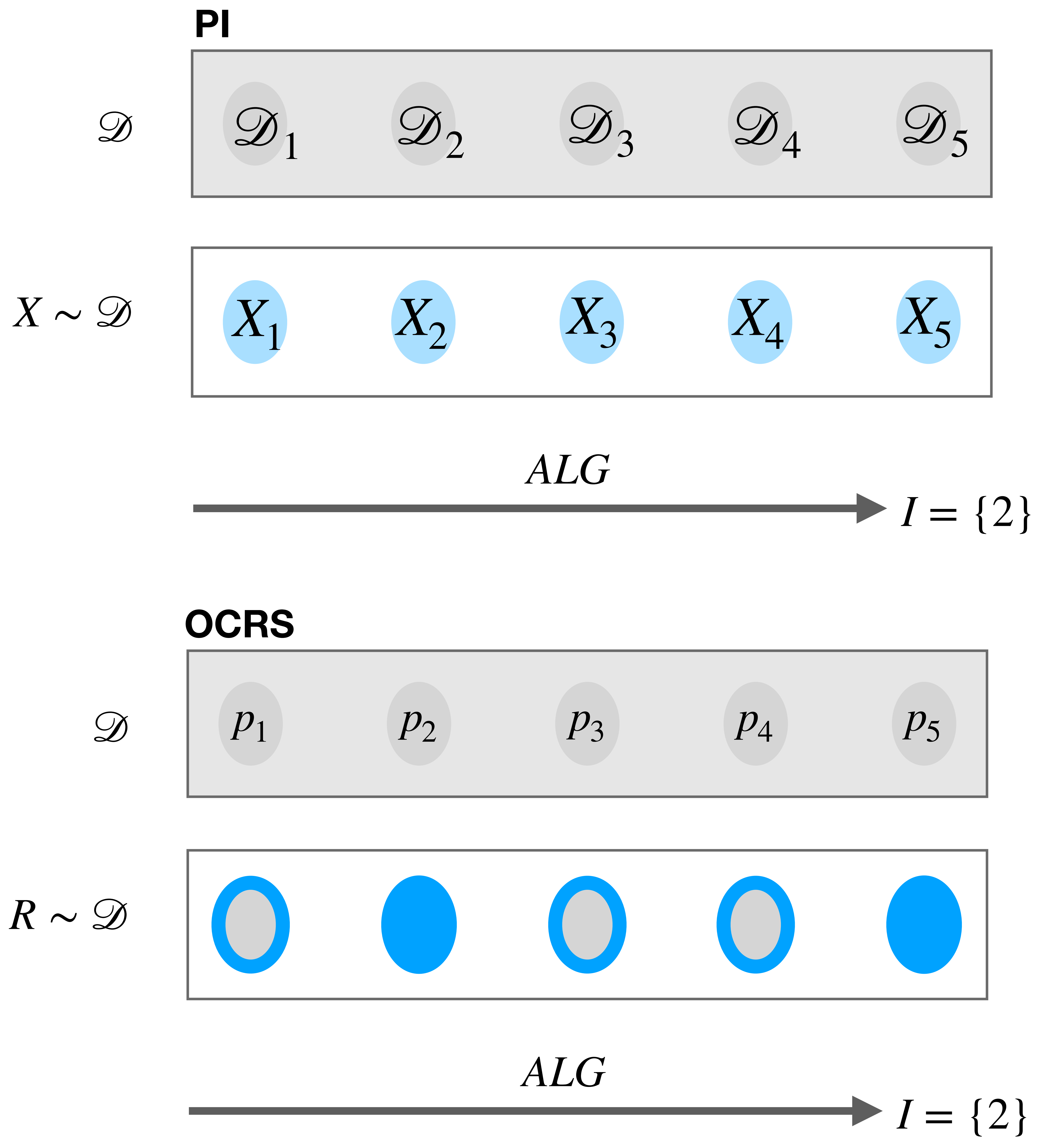

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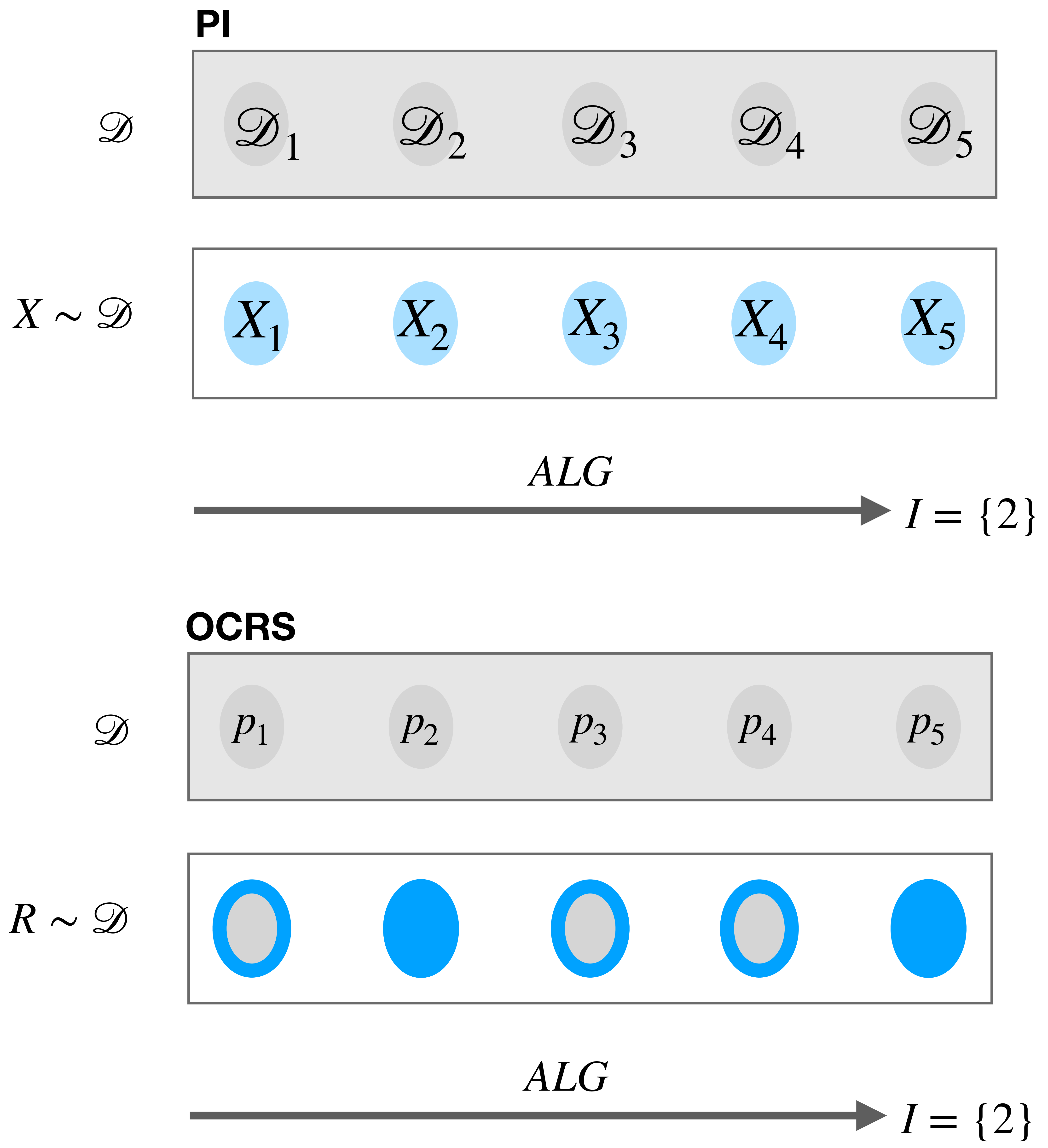

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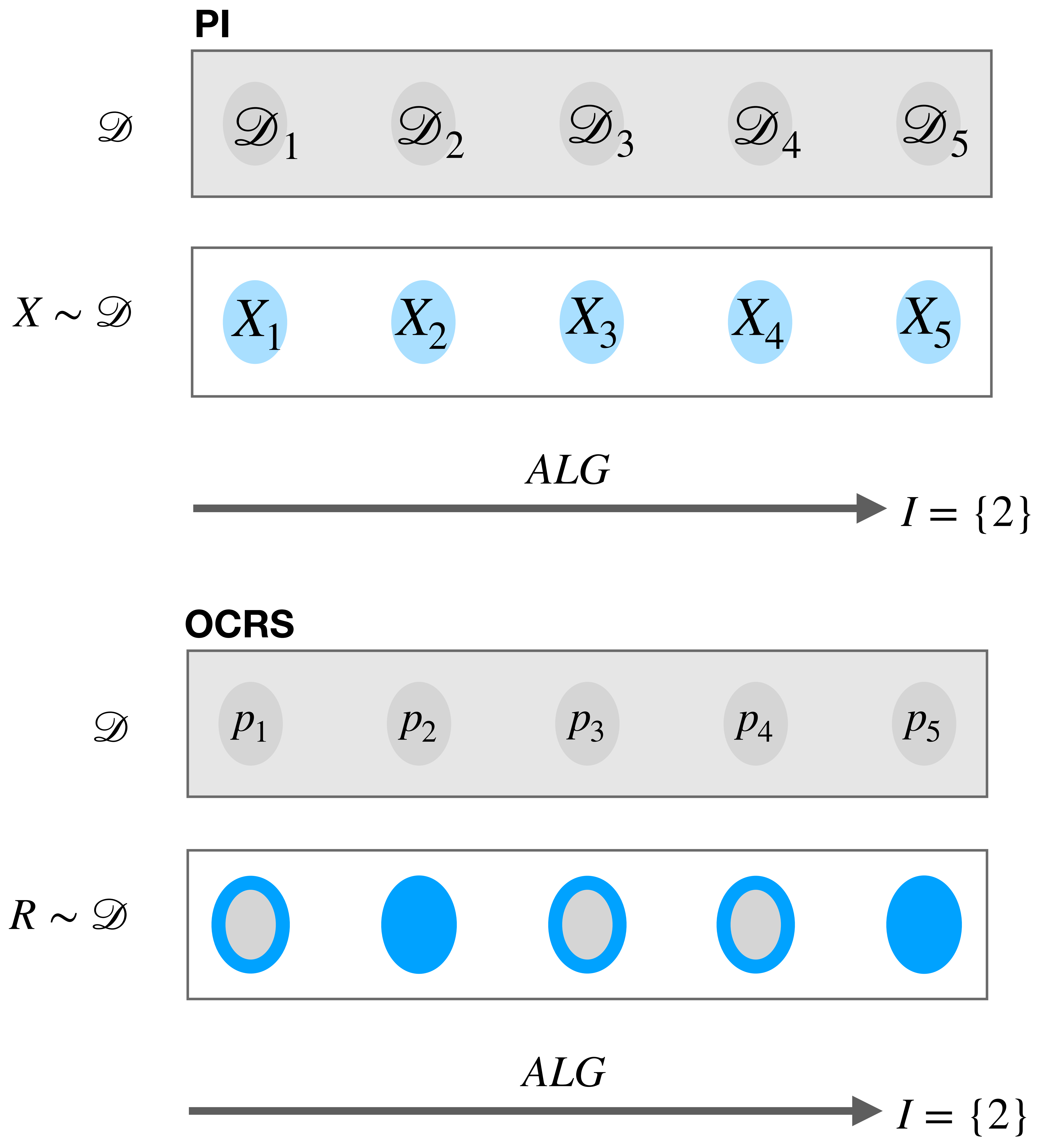

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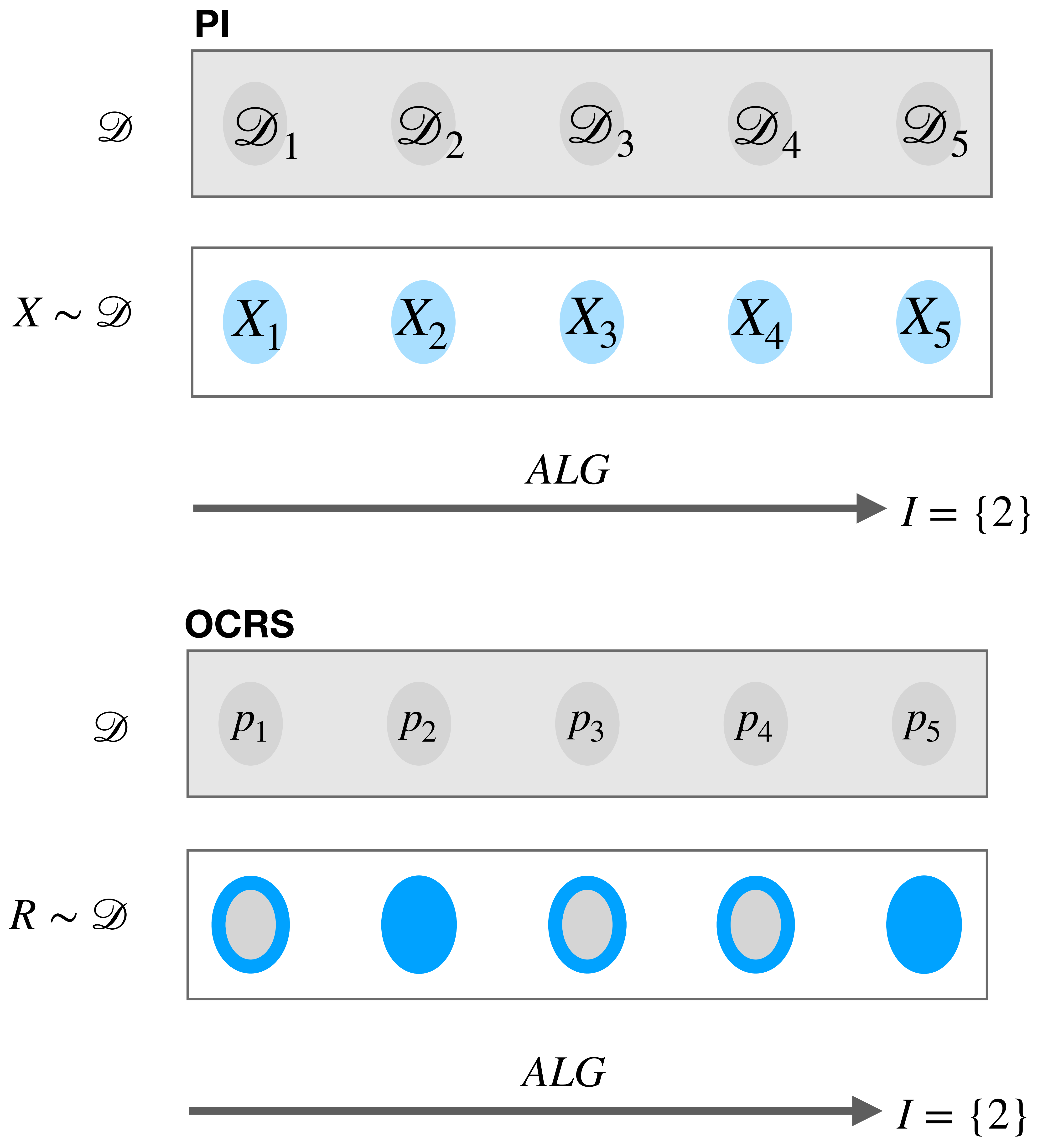

Summary of Results

Pairwise-independent contention resolution.

Matroids:

1-Uniform

k-Uniform

Laminar

(co)Graphic

Transversal

Regular

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1-Uniform	$\sqrt{2}-1 \approx 0.414$
k-Uniform	-
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(co)Graphic	-
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[Caragiannis Gravin Lu Wang '21]

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Pairwise-independent contention resolution.

Matroids:	(offline adv.) prophet (prior):	(offline adv.) prophet:
1-Uniform	$\sqrt{2}-1 \approx 0.414$	$3-\sqrt{5}-\ln 2 \approx 0.071$ s.s.
k-Uniform	-	$1-O(k^{-1/5})$
Laminar	-	$\Omega(1)$
(co)Graphic	-	$\Omega(1)$
Transversal	-	$\Omega(1)$
Regular	-	$\Omega(1)$
Binary	-	-
Linear	-	-
…		

[Caragiannis Gravin Lu Wang '21]

Summary of Results

Pairwise-independent contention resolution.

Matroids:	(offline adv.) prophet (prior):	(offline adv.) prophet:	(almighty adv.) OCRS:
1-Uniform	$\sqrt{2}-1 \approx 0.414$	$3-\sqrt{5}-\ln 2 \approx 0.071$ s.s.	$\leq 1/4$
k-Uniform	-	$1-O(k^{-1/5})$	$1-O(k^{-1/5})$
Laminar	-	$\Omega(1)$	$\Omega(1)$
(co)Graphic	-	$\Omega(1)$	$\Omega(1)$
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Regular	-	$\Omega(1)$	$\Omega(1)$
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[Caragiannis Gravin Lu Wang '21]

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WRONG! Not independent!

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Setting $\delta = O(k^{1/3})$, get $(1 - O(k^{-1/3}))$ - OCRS.

Gives CRS but not OCRS, because algorithm needs to choose order.

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Markov — PI — Same lemma as before

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Combining, balancing, setting $b \approx \delta^{-3/2}k^{-1/2}$ and $\delta = k^{-1/5}$, get $1 - O(k^{-1/5})$ OCRS.

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t	1	2	3	4	5	6	$\cdots$
$c(t)$	$\frac{1}{n}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{91}{144}$	$\cdots$

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$$c_n(t) = 1 - \sum_{k=0}^{t} \frac{(-1)^k}{k!} \to 1 - 1/e$$

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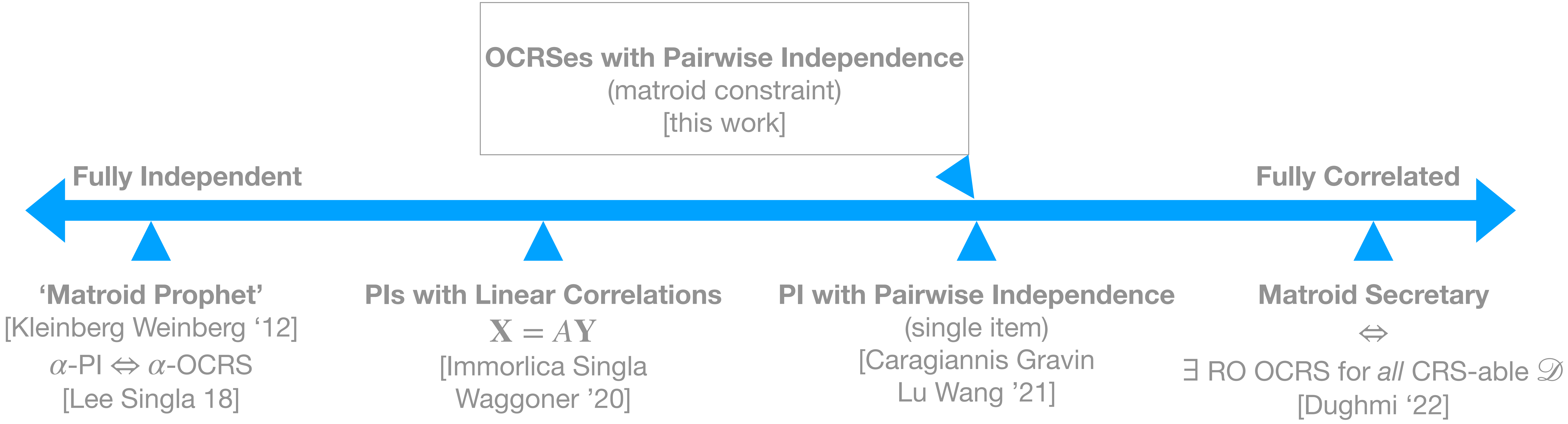
↑ partial sums of Taylor series for $1 - e^{-x}$ at 0!

Limitations

[Dughmi Kalayci Patel ‘24]



Limitations

[Dughmi Kalayci Patel '24]

OCRSes with Pairwise Independence
(matroid constraint)
[this work]

Limitations of Pairwise Independent Stochastic Selection
[Dughmi Kalayci Patel '24]

Fully Independent — **Fully Correlated**

'Matroid Prophet'
[Kleinberg Weinberg '12]
α-PI ⇔ α-OCRS
[Lee Singla 18]

PIs with Linear Correlations
$\mathbf{X} = A\mathbf{Y}$
[Immorlica Singla Waggoner '20]

PI with Pairwise Independence
(single item)
[Caragiannis Gravin Lu Wang '21]

Matroid Secretary
⇔
∃ RO OCRS for *all* CRS-able $\mathscr{D}$
[Dughmi '22]

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Matroid Secretary
$\Leftrightarrow$
$\exists$ RO OCRS for *all* CRS-able $\mathscr{D}$
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Laminar		
(co)Graphic	$\Omega(1)$	$\Omega(1)$
Transversal		
Regular		
Binary	$O(1/r)$	
Linear		$O(1/\log r)$
...		

$r =$ Rank$(\mathscr{M})$
vs. even the non-adaptive adversary

In Parting…

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3. What is the constant for the single-item pairwise-independent prophet inequality?

Thank you!

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