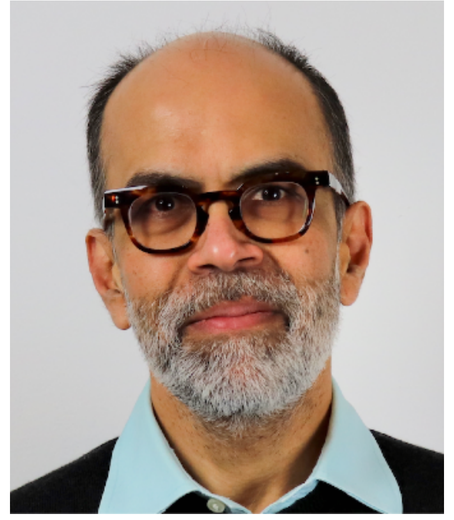
Set Covering with Our Eyes Wide Shut



Anupam Gupta (NYU)

Gregory Kehne (UT Austin)

Roie Levin (Rutgers)

January 10, 2024

SODA 2024

Online Packing

Online Packing

$$\max \; c \cdot x$$
$$1 \cdot x \leq 1$$
$$x \in \{0,1\}^m$$

Online Packing

$$\max \ c \cdot x$$
$$1 \cdot x \leq 1$$
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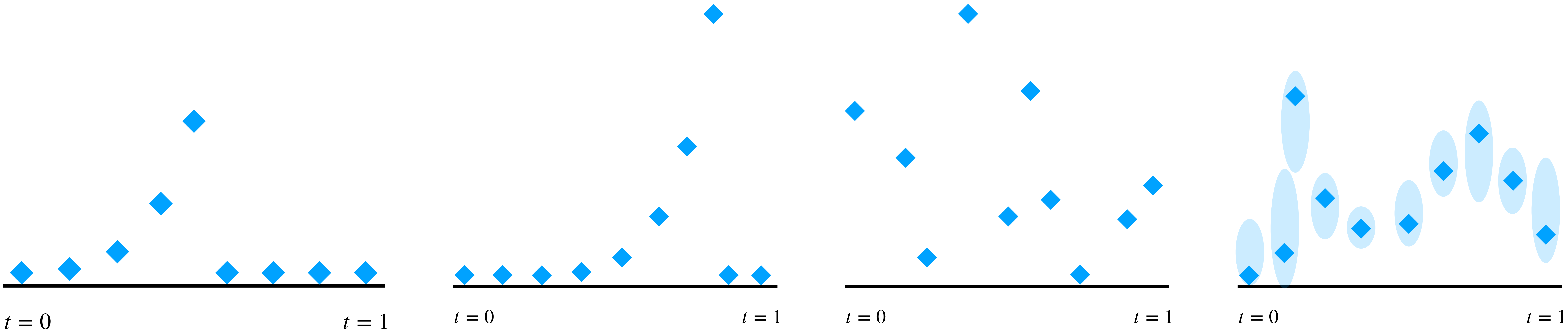


adversarial sequence

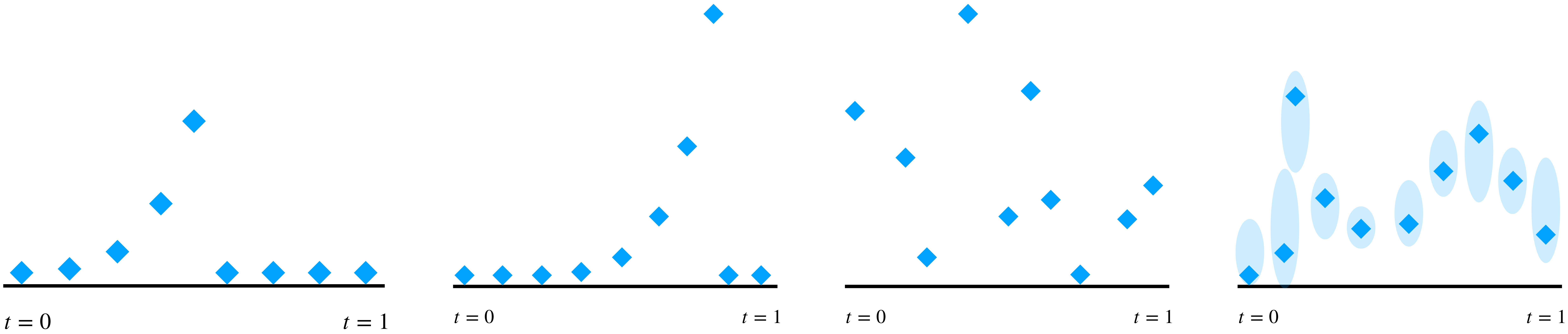
RO (secretary)

Prophet

Online Packing

$$\max \ c \cdot x$$
$$1 \cdot x \leq 1$$
$$x \in \{0,1\}^m$$

- c_i arrive online



adversarial sequence RO (secretary) Prophet

Online Packing

$$\max \ c \cdot x$$
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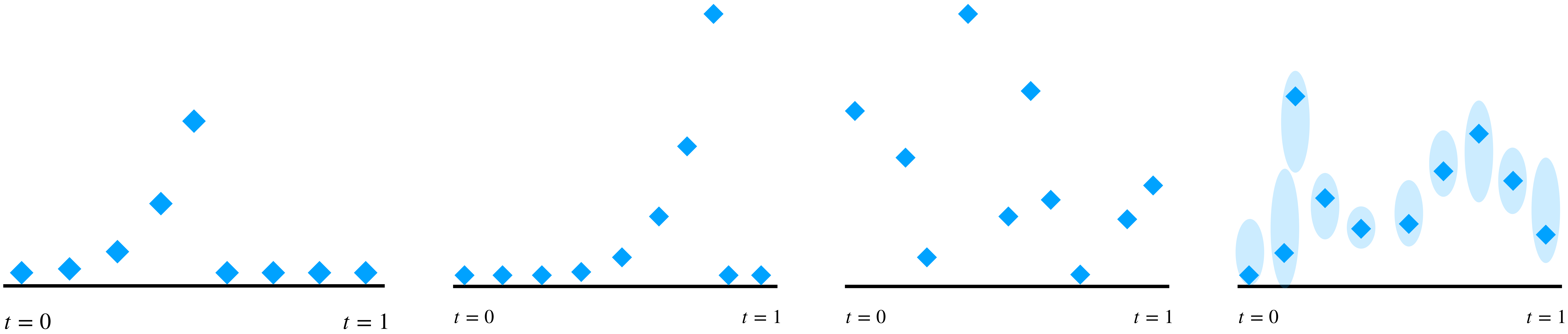
- c_i arrive online
- adversarial instance + order: intractable



adversarial sequence

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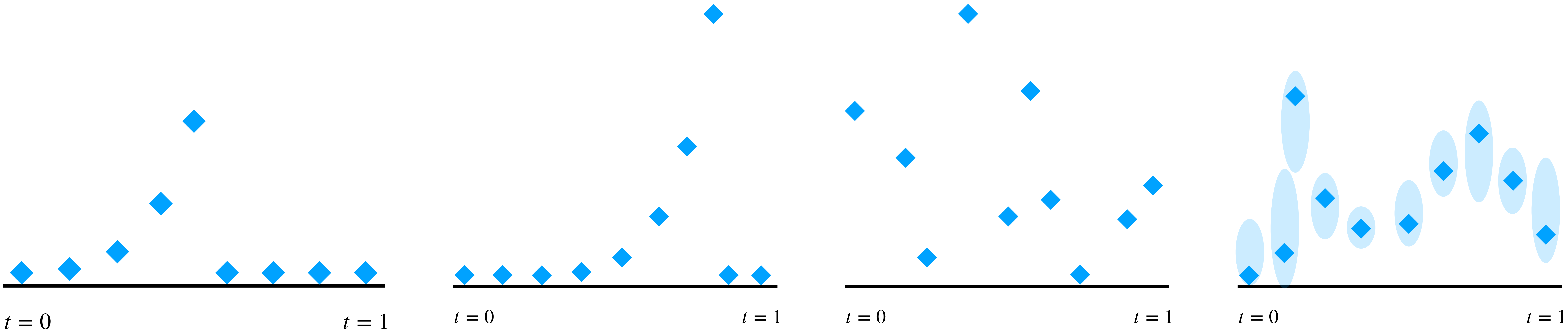
Online Packing

$$\max \; c \cdot x$$
$$1 \cdot x \leq 1$$
$$x \in \{0,1\}^m$$

- c_i arrive online
- adversarial instance + order: intractable
- RO, stochastic: tractable $(1/e,\ 1/2)$



adversarial sequence

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Online Covering

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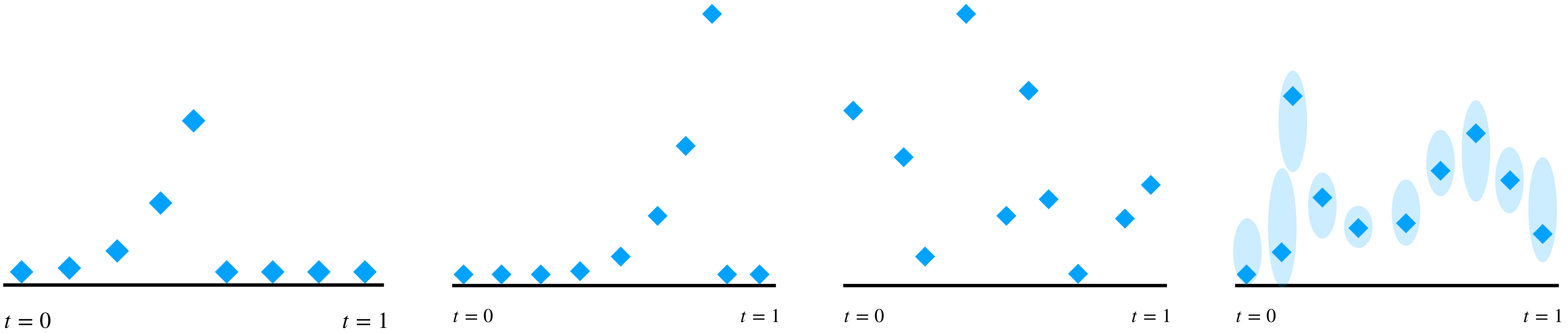


adversarial sequence

RO (secretary)

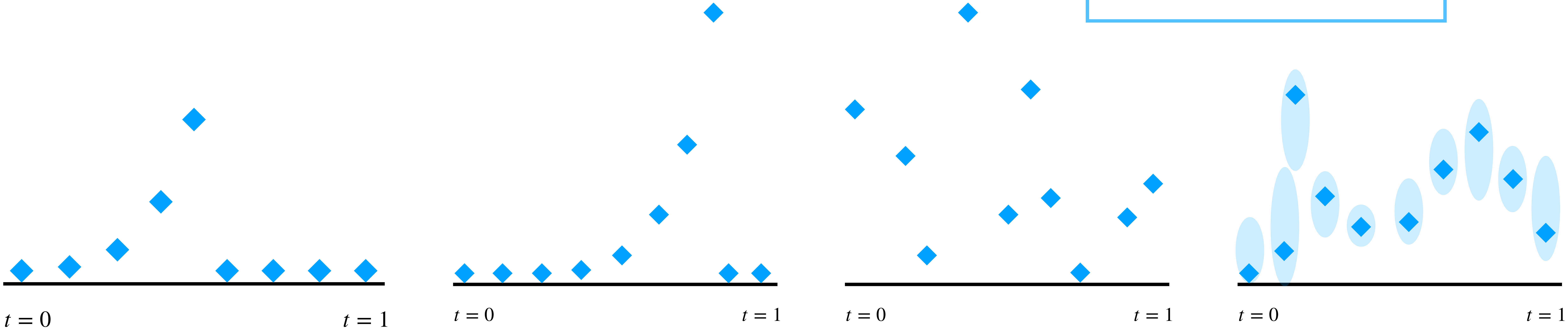
Prophet

Online Packing

Online Covering

$$\max \ c \cdot x$$
$$1 \cdot x \leq 1$$
$$x \in \{0,1\}^m$$

$$\min \ c \cdot x$$
$$A_1 \cdot x \geq 1$$
$$\vdots$$
$$A_n \cdot x \geq 1$$
$$x \in \{0,1\}^m$$



adversarial sequence

RO (secretary)

Prophet

Online Packing

Online Covering

$$\max\ c \cdot x$$
$$1 \cdot x \leq 1$$
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- **constraints** arrive online

$$\min\ c \cdot x$$
$$A_1 \cdot x \geq 1$$
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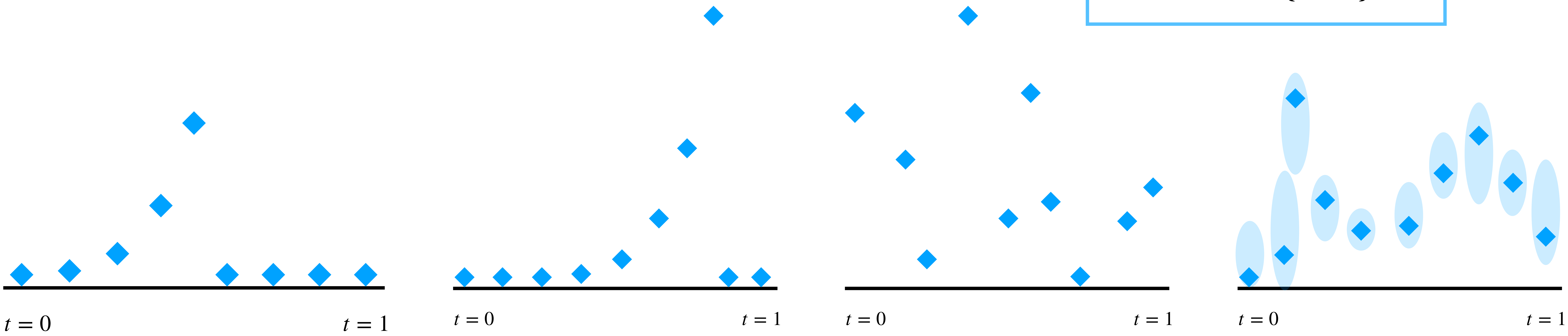


adversarial sequence

RO (secretary)

Prophet

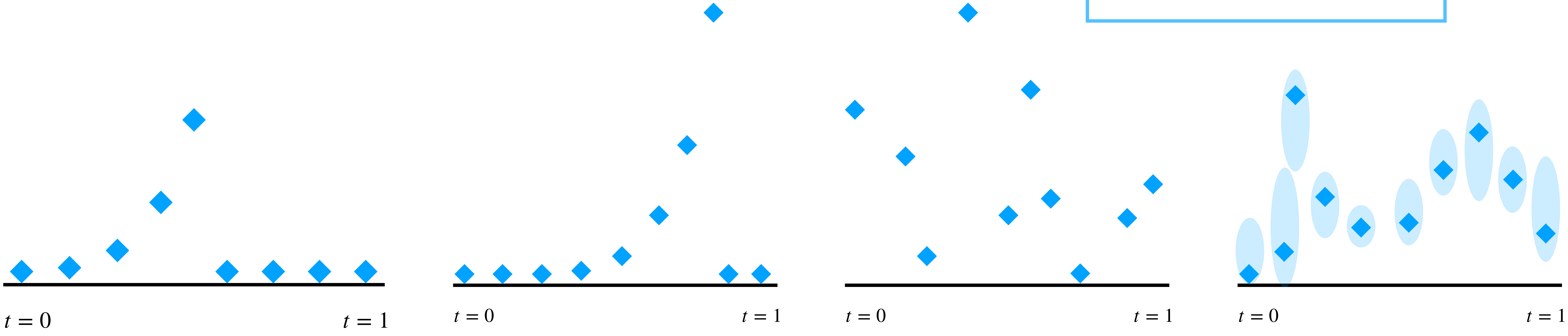
Online Packing

Online Covering

$$\max \ c \cdot x$$
$$1 \cdot x \leq 1$$
$$x \in \{0,1\}^m$$

- **constraints** arrive online
- c, x known at the outset

$$\min \ c \cdot x$$
$$A_1 \cdot x \geq 1$$
$$\vdots$$
$$A_n \cdot x \geq 1$$
$$x \in \{0,1\}^m$$

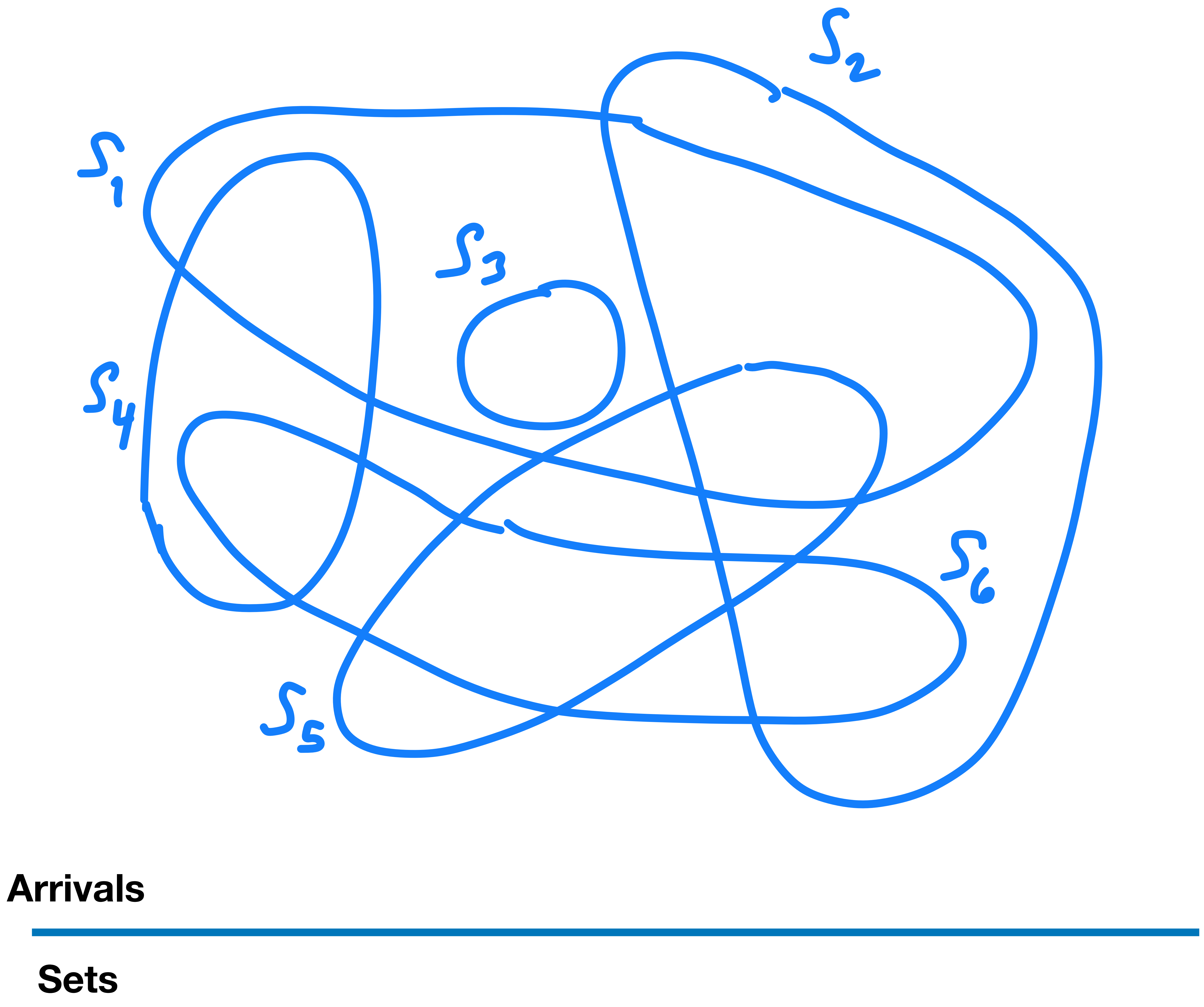


adversarial sequence

RO (secretary)

Prophet

Online Set Cover



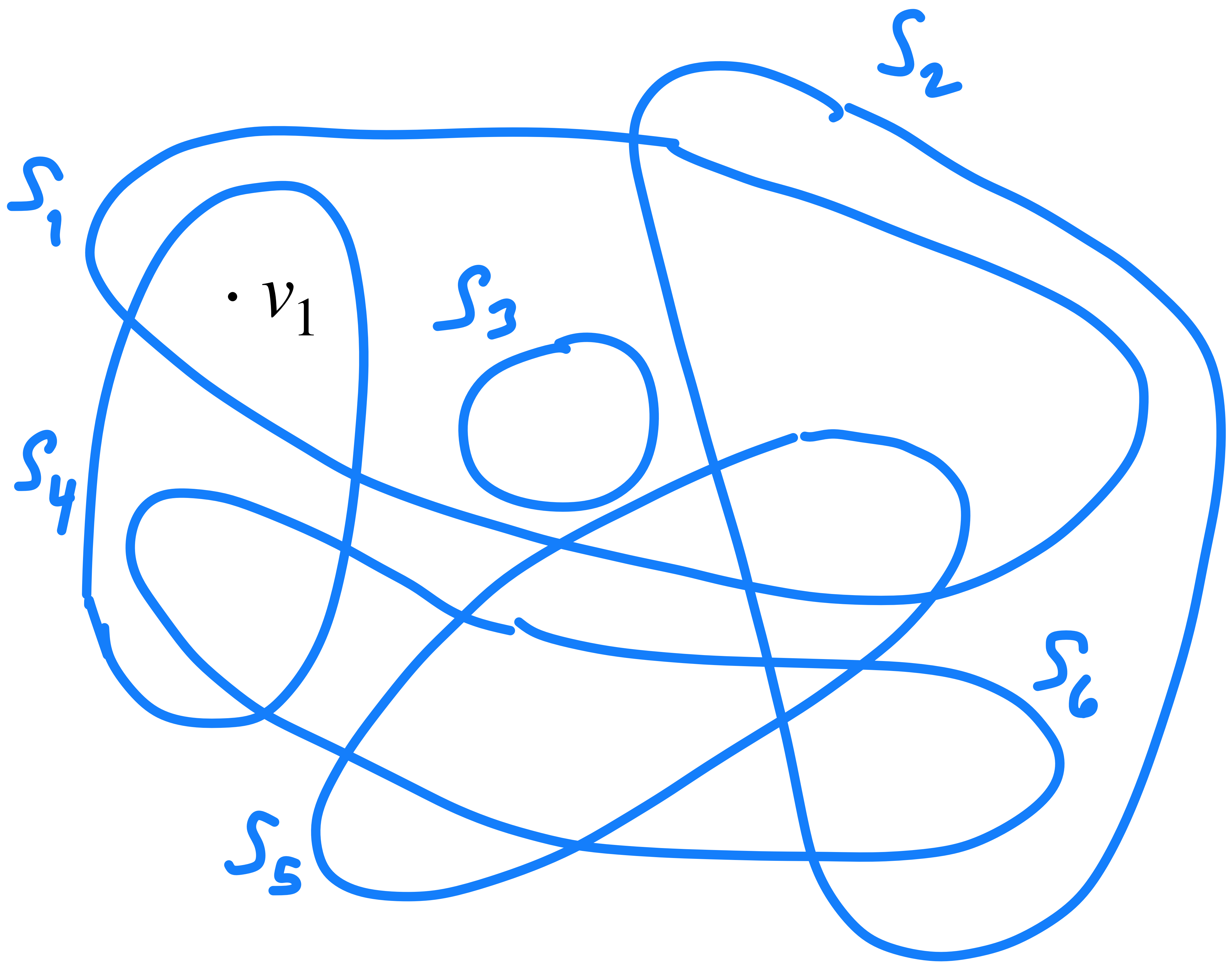
Online Set Cover:

$$
\begin{aligned}
\min\ & c \cdot x \\
A_1 \cdot x &\geq 1 \\
&\vdots \\
A_n \cdot x &\geq 1 \\
x &\in \{0,1\}^m
\end{aligned}
$$

Goals:

- Satisfy constraints online
- Buy each x_S irrevocably
- Compete with offline OPT

Online Set Cover



Arrivals $\cdot\, v_1$

Sets

Online Set Cover:

$$\min\ c \cdot x$$
$$A_1 \cdot x \geq 1$$
$$\vdots$$
$$A_n \cdot x \geq 1$$
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Goals:

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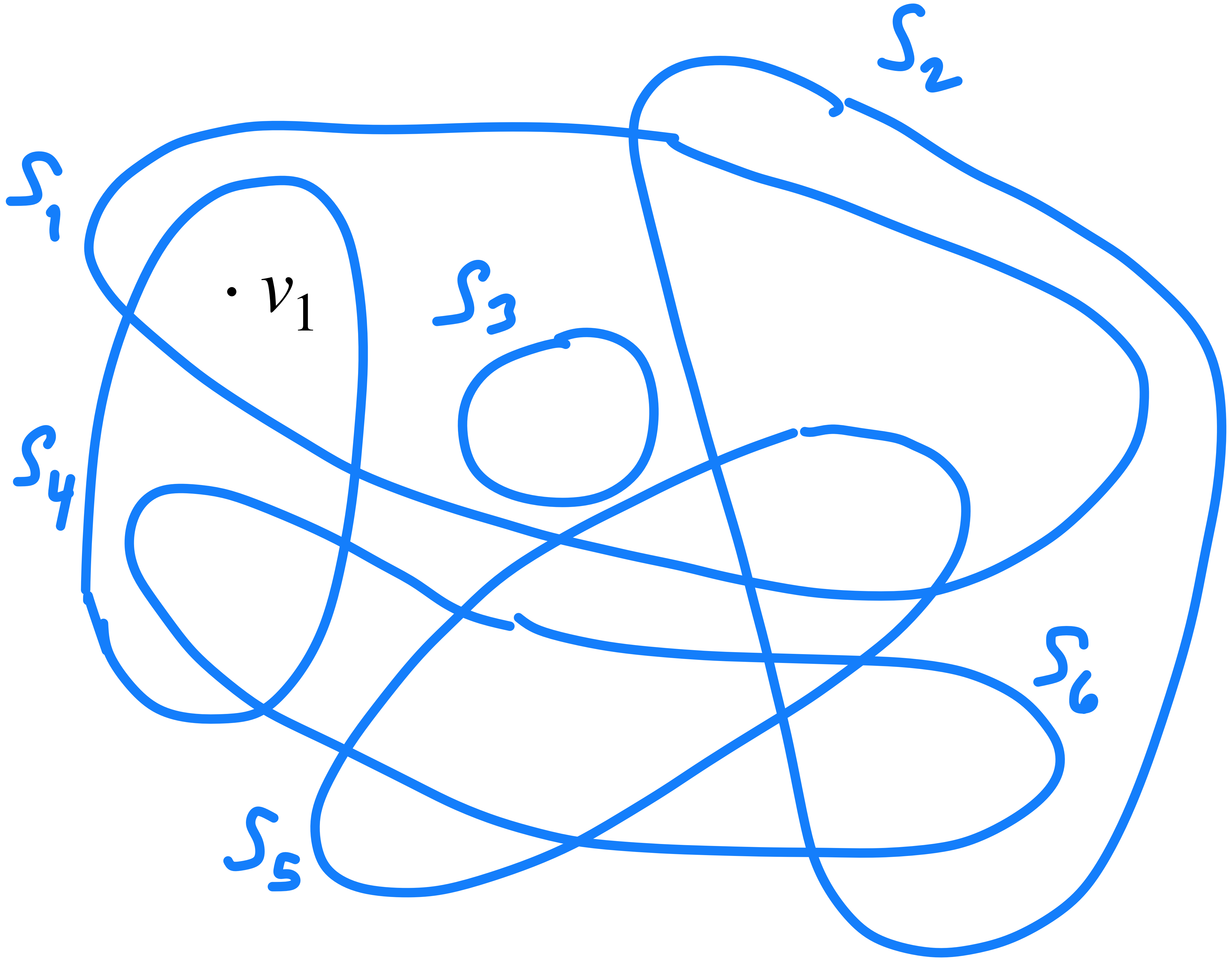
Online Set Cover



Arrivals $\cdot\, v_1$

Sets S_4

Online Set Cover:

$$\begin{aligned} \min\ & c \cdot x \\ A_1 \cdot x &\geq 1 \\ &\vdots \\ A_n \cdot x &\geq 1 \\ x &\in \{0,1\}^m \end{aligned}$$

Goals:

- Satisfy constraints online
- Buy each x_S irrevocably
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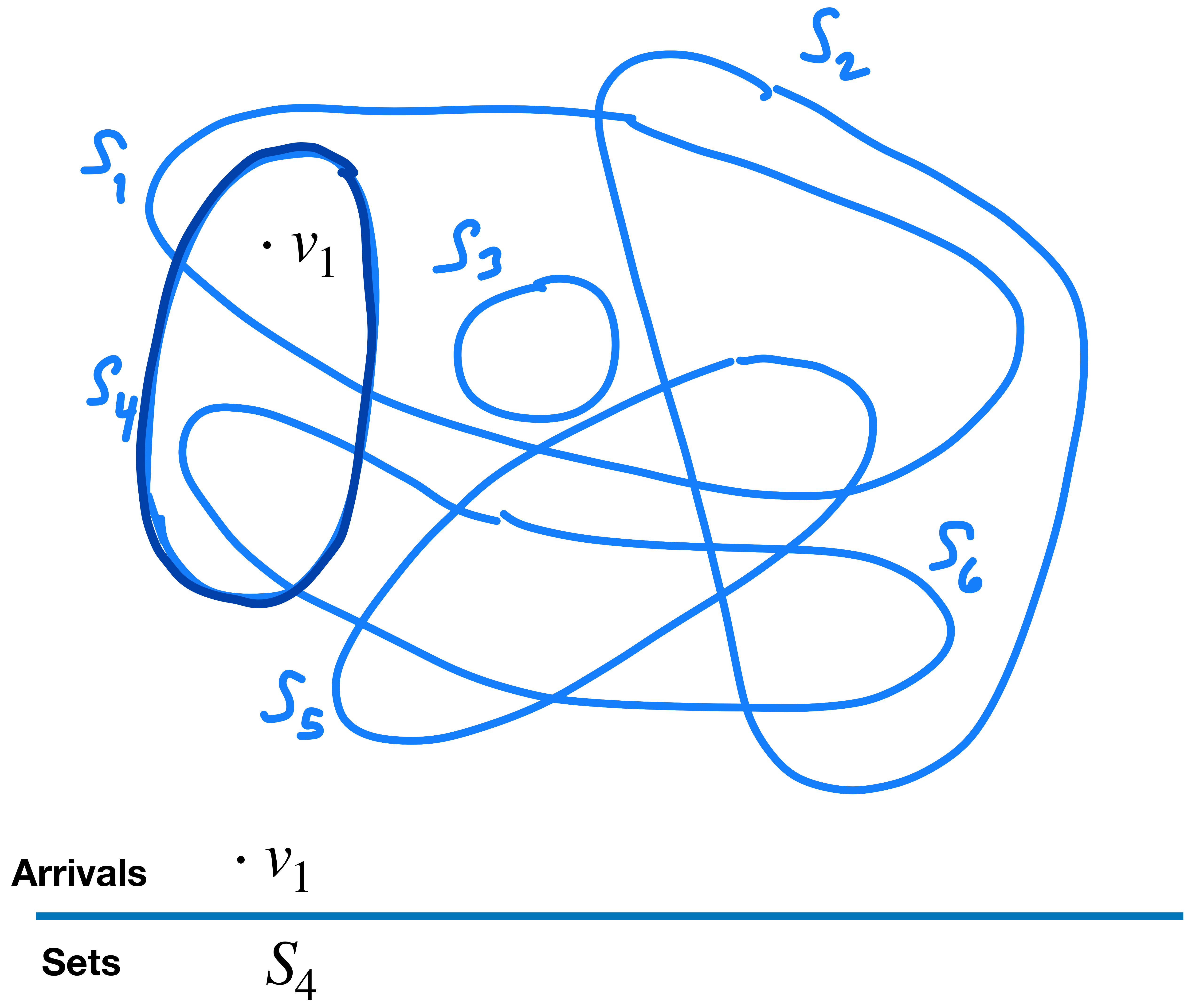
Online Set Cover



Online Set Cover:

$$\min \; c \cdot x$$
$$A_1 \cdot x \geq 1$$
$$\vdots$$
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Goals:

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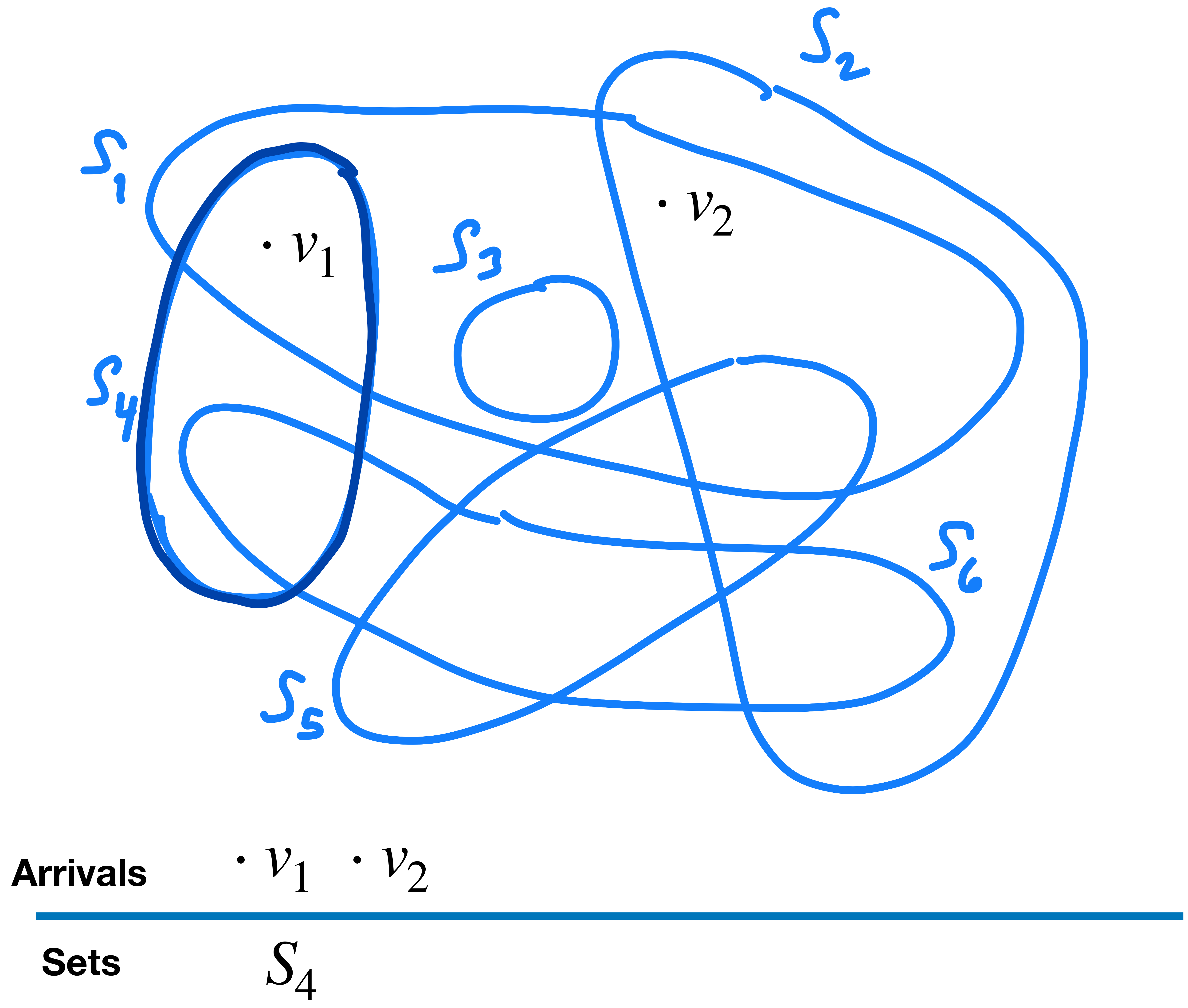
Online Set Cover



Online Set Cover:

$$\begin{aligned} &\min \ c \cdot x \\ &A_1 \cdot x \geq 1 \\ &\vdots \\ &A_n \cdot x \geq 1 \\ &x \in \{0,1\}^m \end{aligned}$$

Goals:

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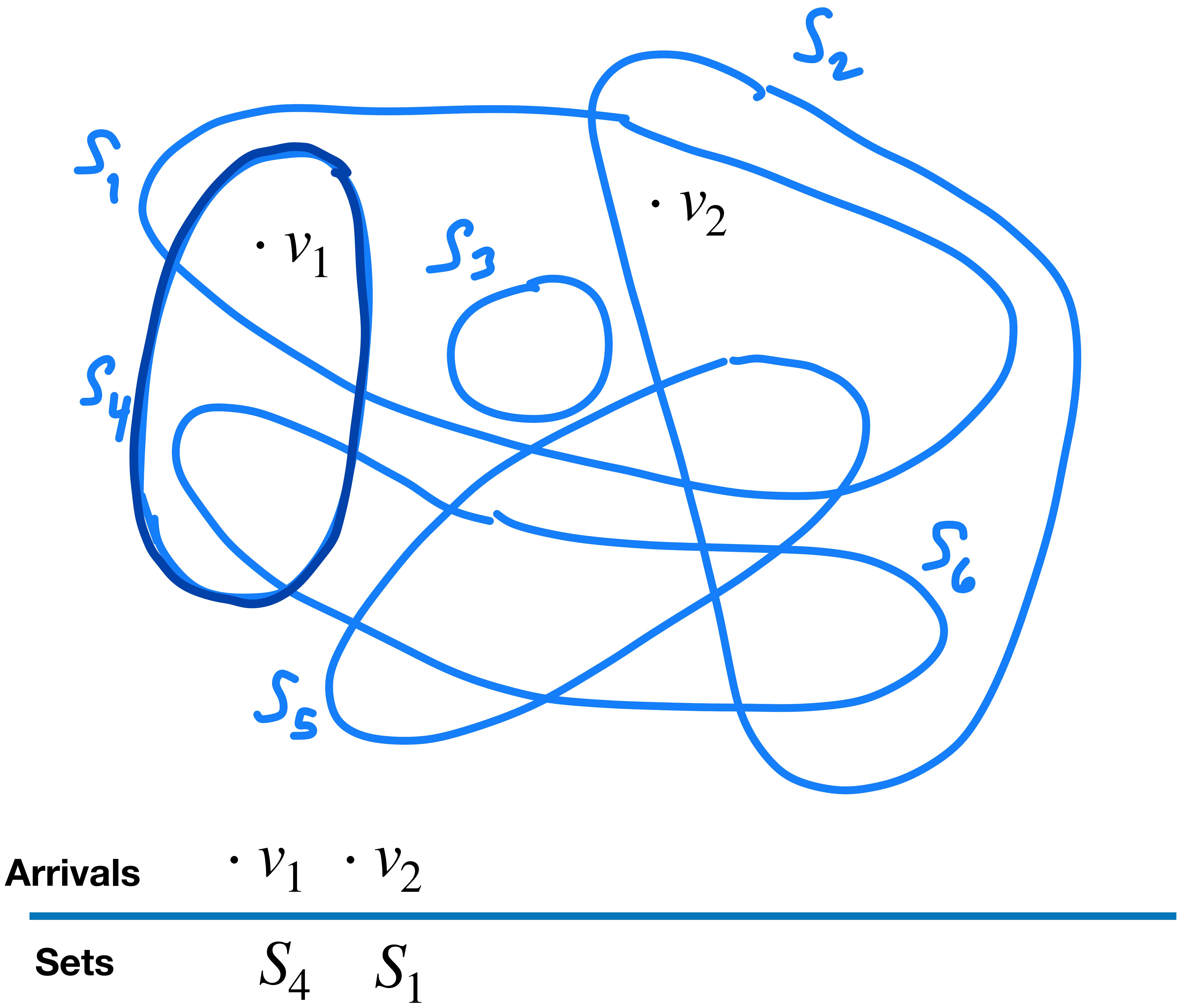
Online Set Cover



Online Set Cover:

$$\min\ c \cdot x$$
$$A_1 \cdot x \geq 1$$
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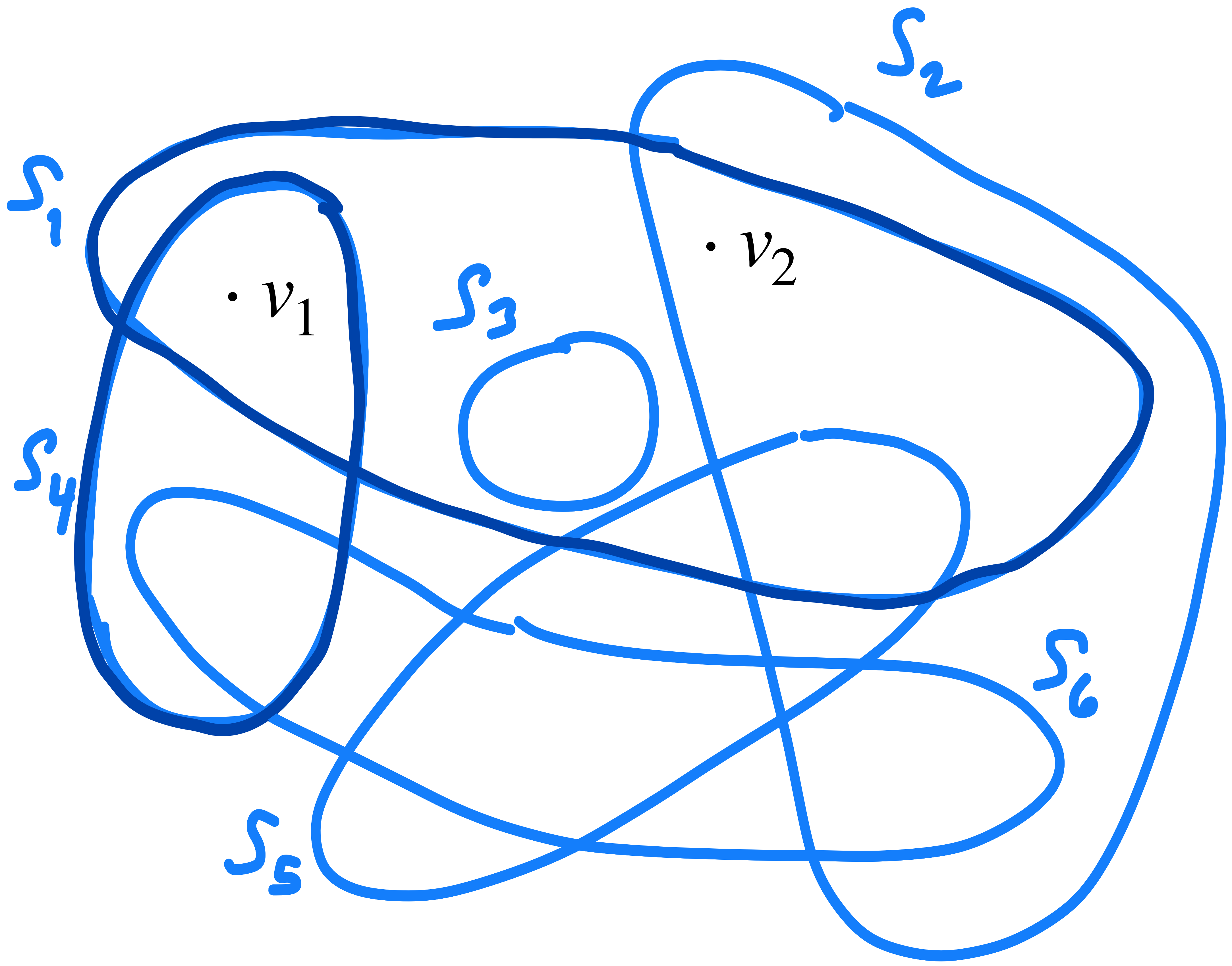

Arrivals	$\cdot\, v_1$	$\cdot\, v_2$
Sets	S_4	S_1

Online Set Cover:

$$\min \; c \cdot x$$
$$A_1 \cdot x \geq 1$$
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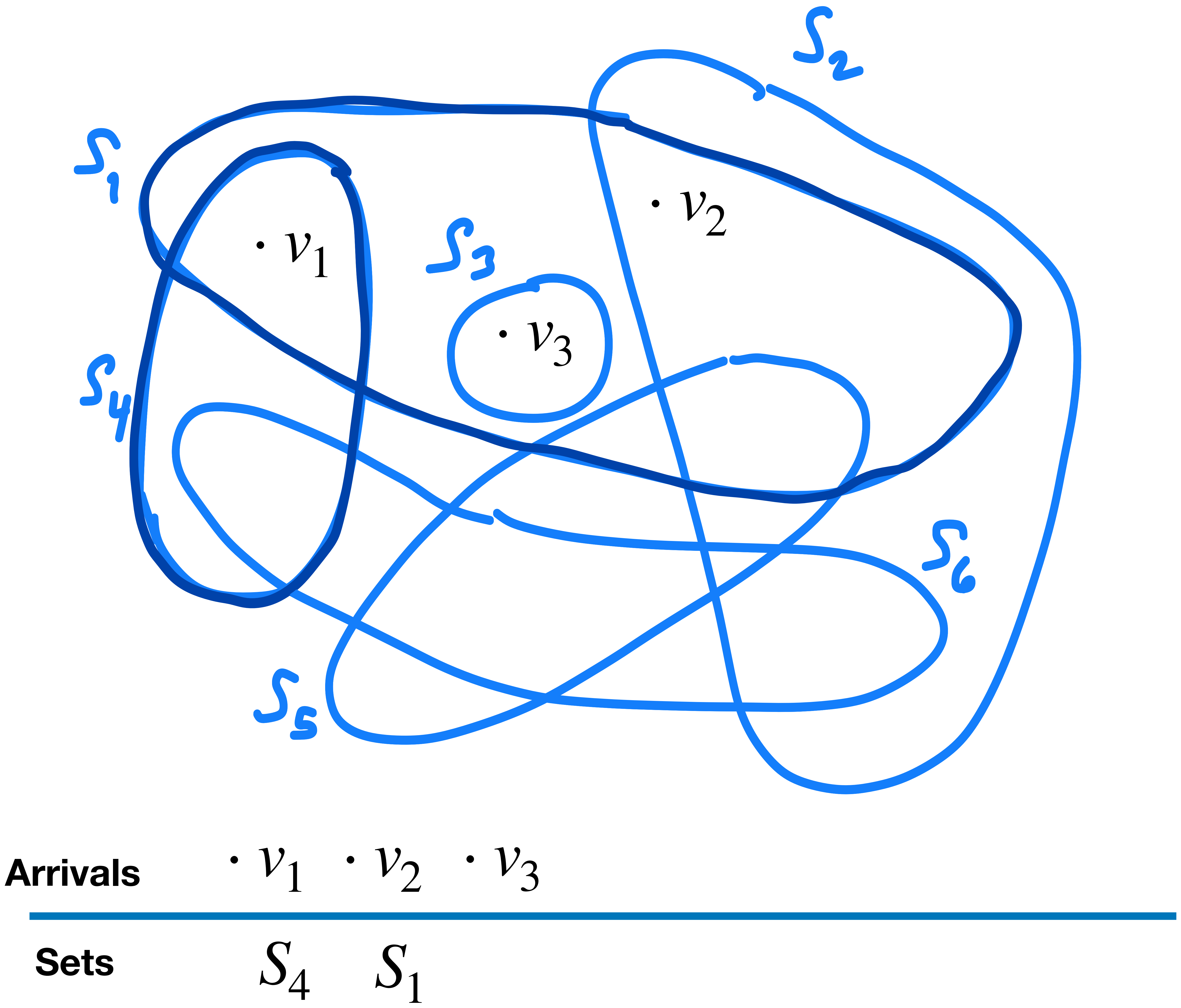
Online Set Cover



Online Set Cover:

$$
\begin{aligned}
\min\ & c \cdot x \\
A_1 \cdot x &\geq 1 \\
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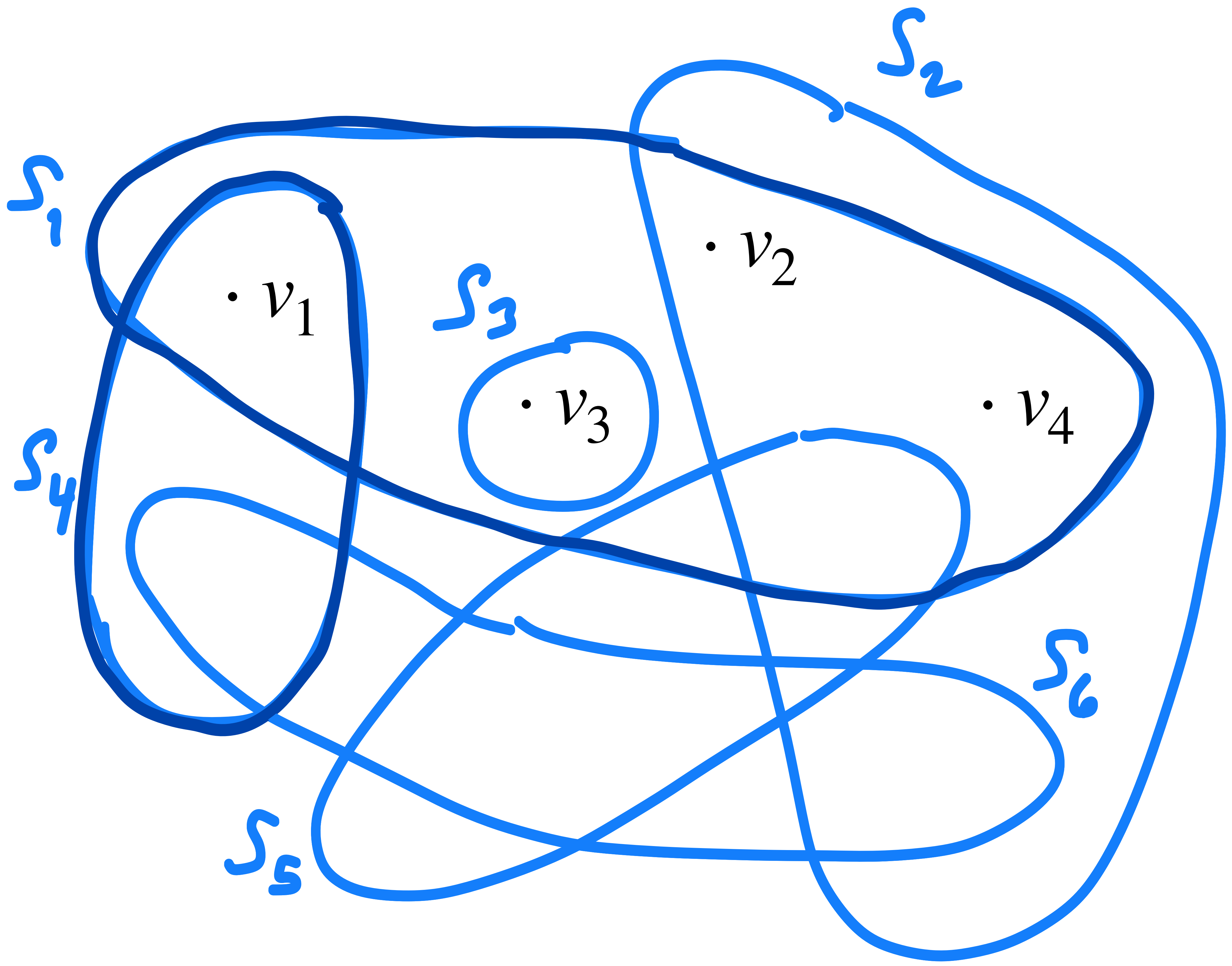
Online Set Cover



Arrivals $\cdot\, v_1$ $\cdot\, v_2$ $\cdot\, v_3$ $\cdot\, v_4$

Sets S_4 S_1

Online Set Cover:

$$\begin{aligned} &\min\ c \cdot x \\ &A_1 \cdot x \geq 1 \\ &\quad \vdots \\ &A_n \cdot x \geq 1 \\ &\quad x \in \{0,1\}^m \end{aligned}$$

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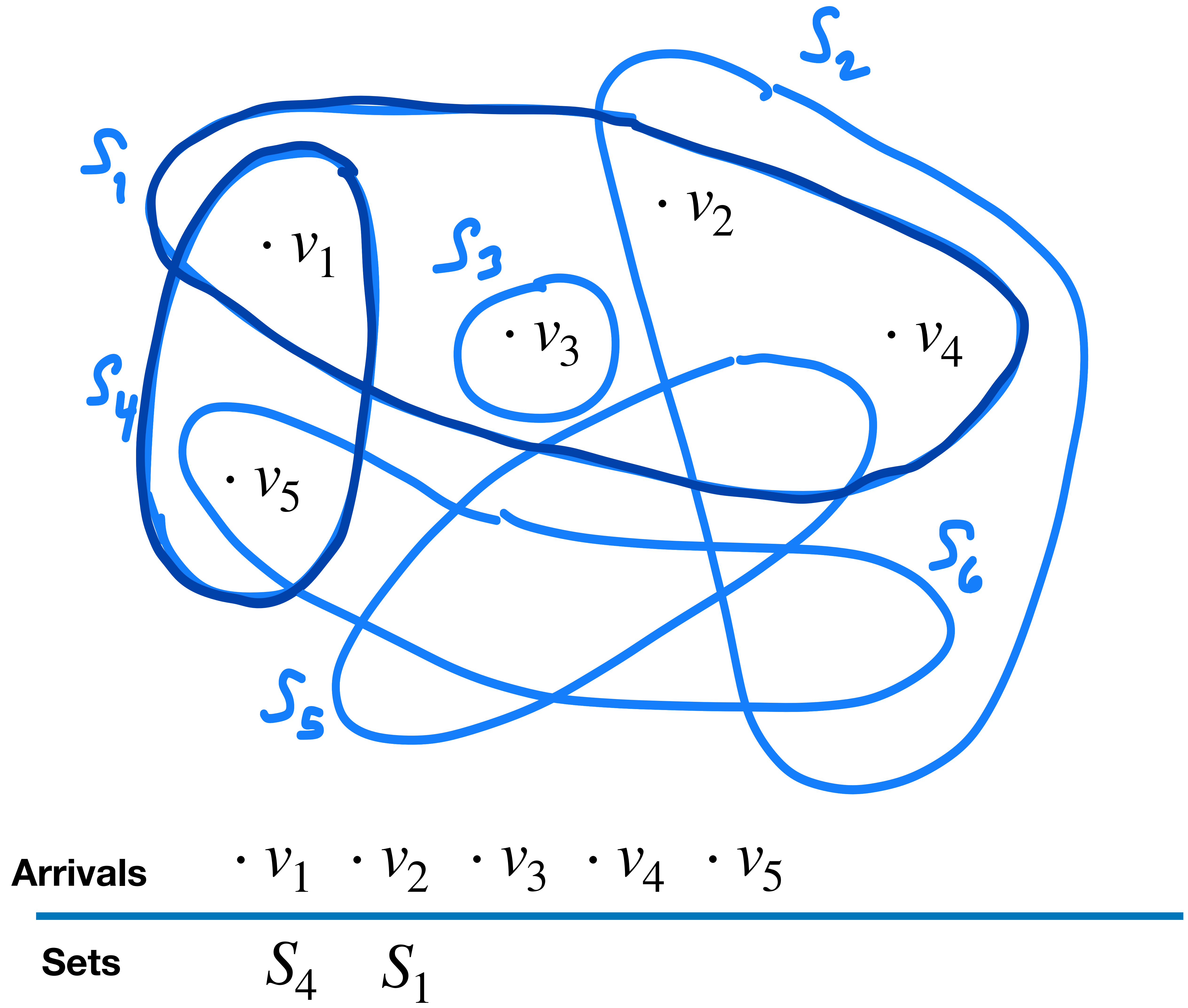
Online Set Cover



Online Set Cover:

$$
\begin{aligned}
\min\ & c \cdot x \\
A_1 \cdot x & \geq 1 \\
& \vdots \\
A_n \cdot x & \geq 1 \\
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\end{aligned}
$$

Goals:

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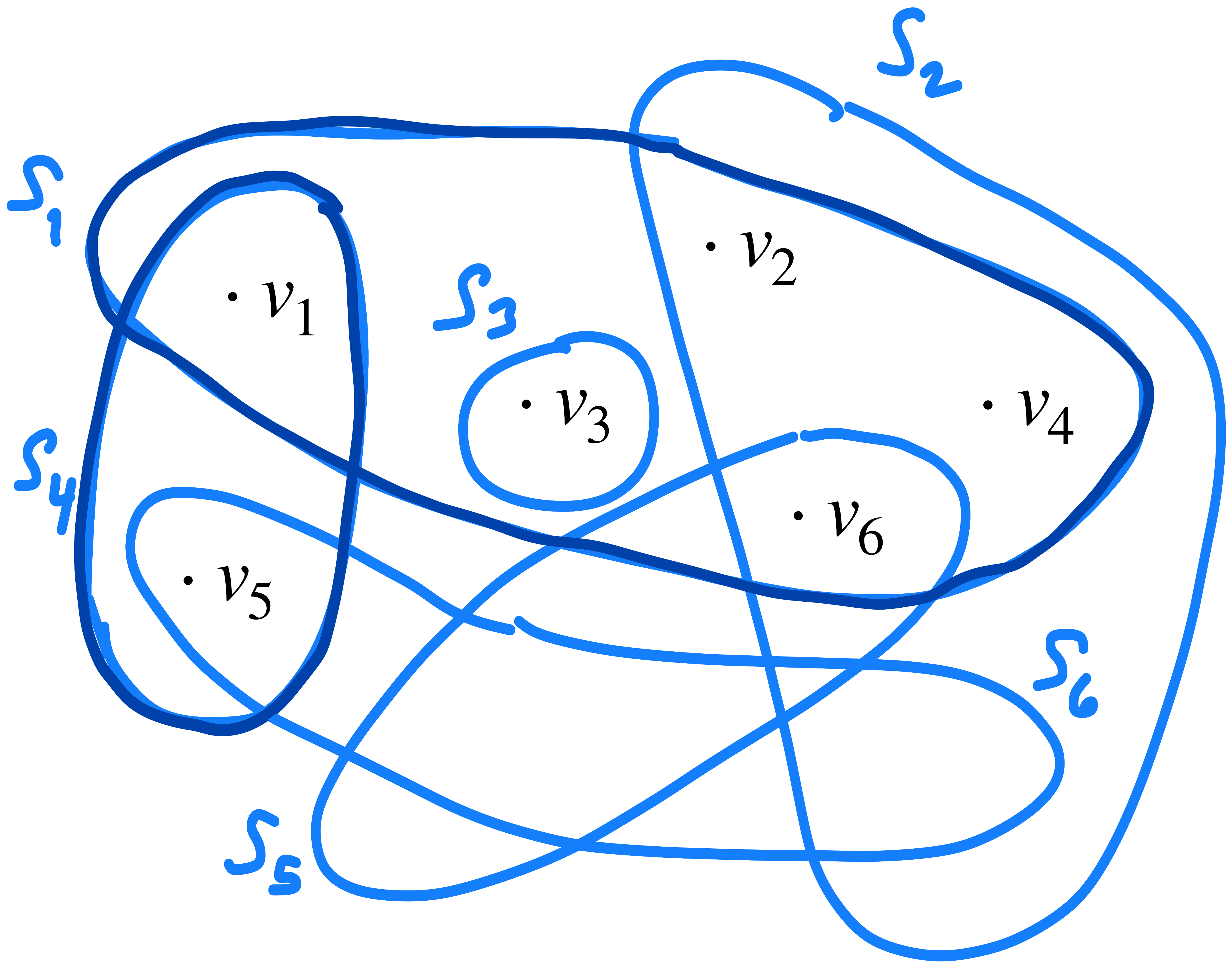
Online Set Cover



Arrivals $\cdot v_1$ $\cdot v_2$ $\cdot v_3$ $\cdot v_4$ $\cdot v_5$ $\cdot v_6$

Sets S_4 S_1

Online Set Cover:

$$\begin{aligned} \min\ & c \cdot x \\ A_1 \cdot x &\geq 1 \\ &\vdots \\ A_n \cdot x &\geq 1 \\ x &\in \{0,1\}^m \end{aligned}$$

Goals:

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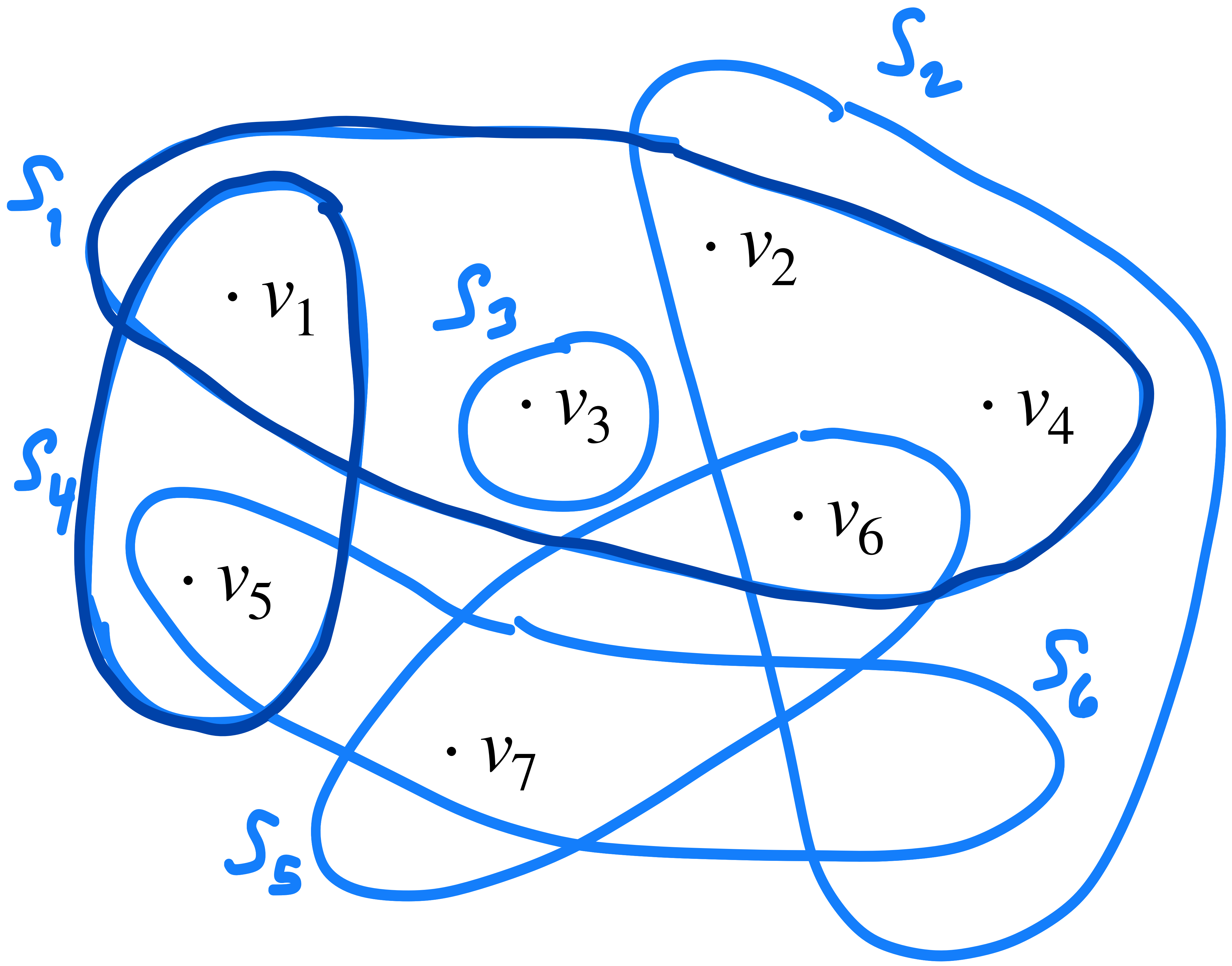
Online Set Cover



Arrivals $\cdot v_1 \quad \cdot v_2 \quad \cdot v_3 \quad \cdot v_4 \quad \cdot v_5 \quad \cdot v_6 \quad \cdot v_7$

Sets $S_4 \quad S_1$

Online Set Cover:

$$\begin{aligned} \min \ & c \cdot x \\ A_1 \cdot x & \geq 1 \\ & \vdots \\ A_n \cdot x & \geq 1 \\ x & \in \{0,1\}^m \end{aligned}$$

Goals:

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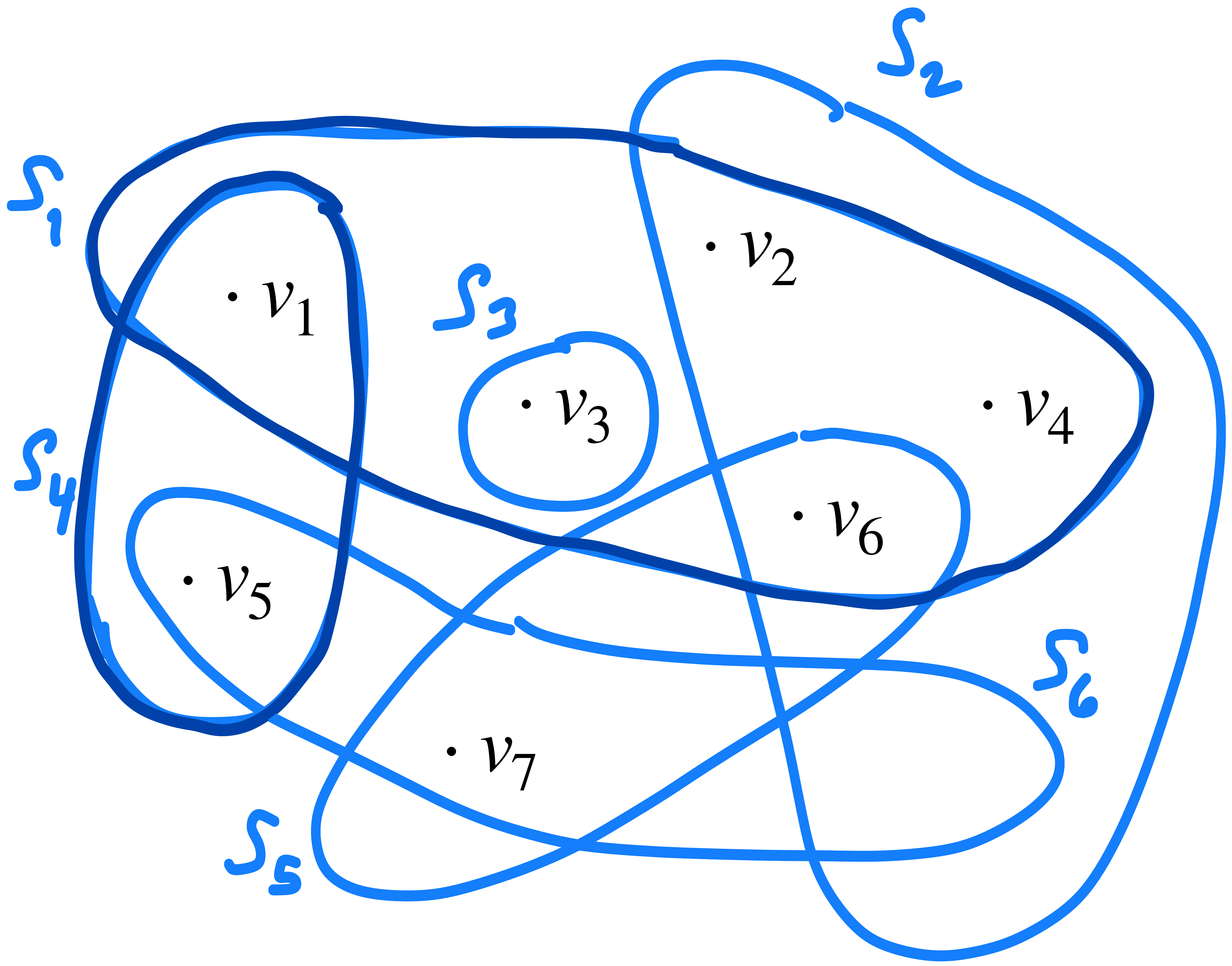
Online Set Cover



Arrivals	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Sets	S_4	S_1					S_5

Online Set Cover:

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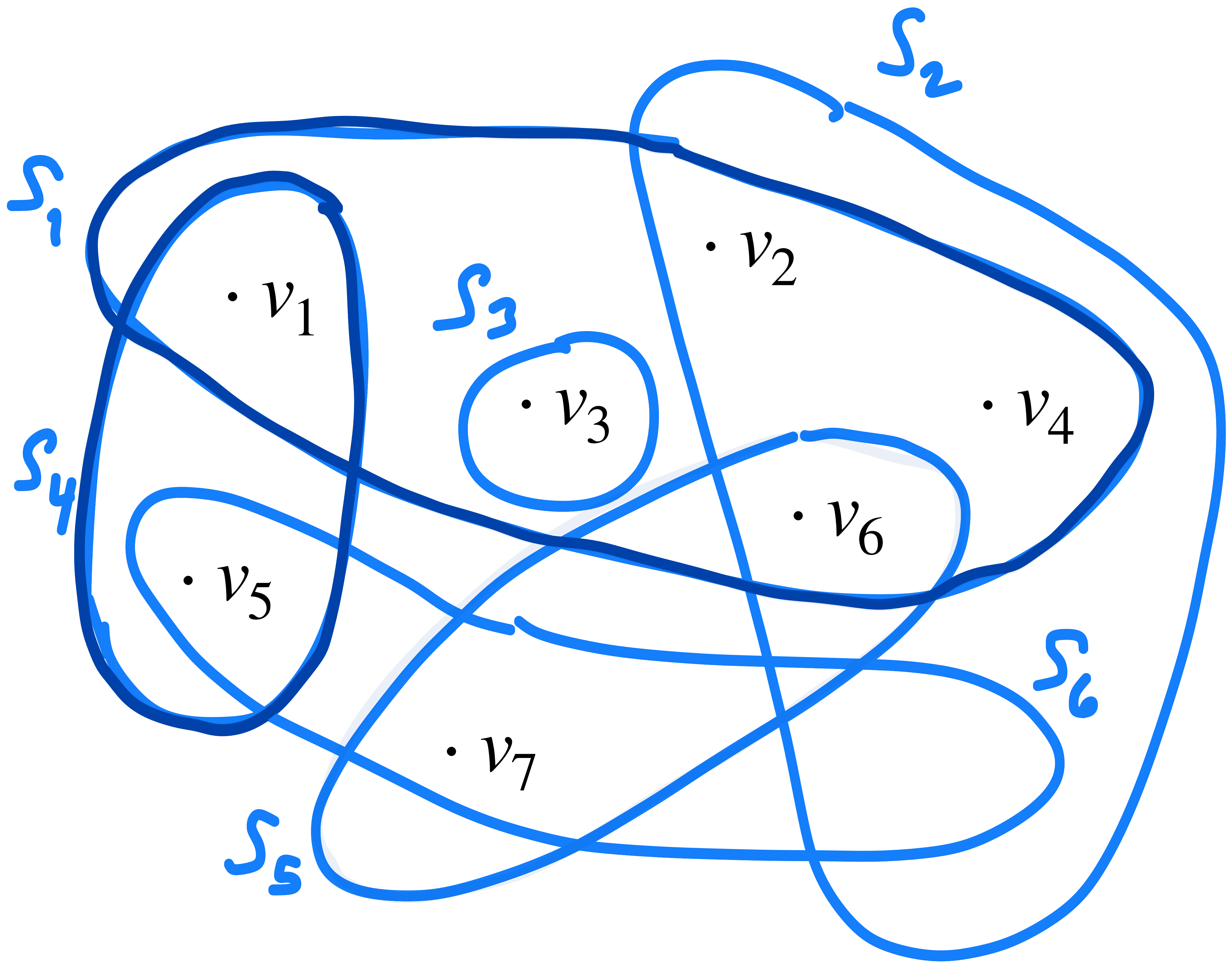

Arrivals $\cdot\, v_1$ $\cdot\, v_2$ $\cdot\, v_3$ $\cdot\, v_4$ $\cdot\, v_5$ $\cdot\, v_6$ $\cdot\, v_7$

Sets S_4 S_1 S_5

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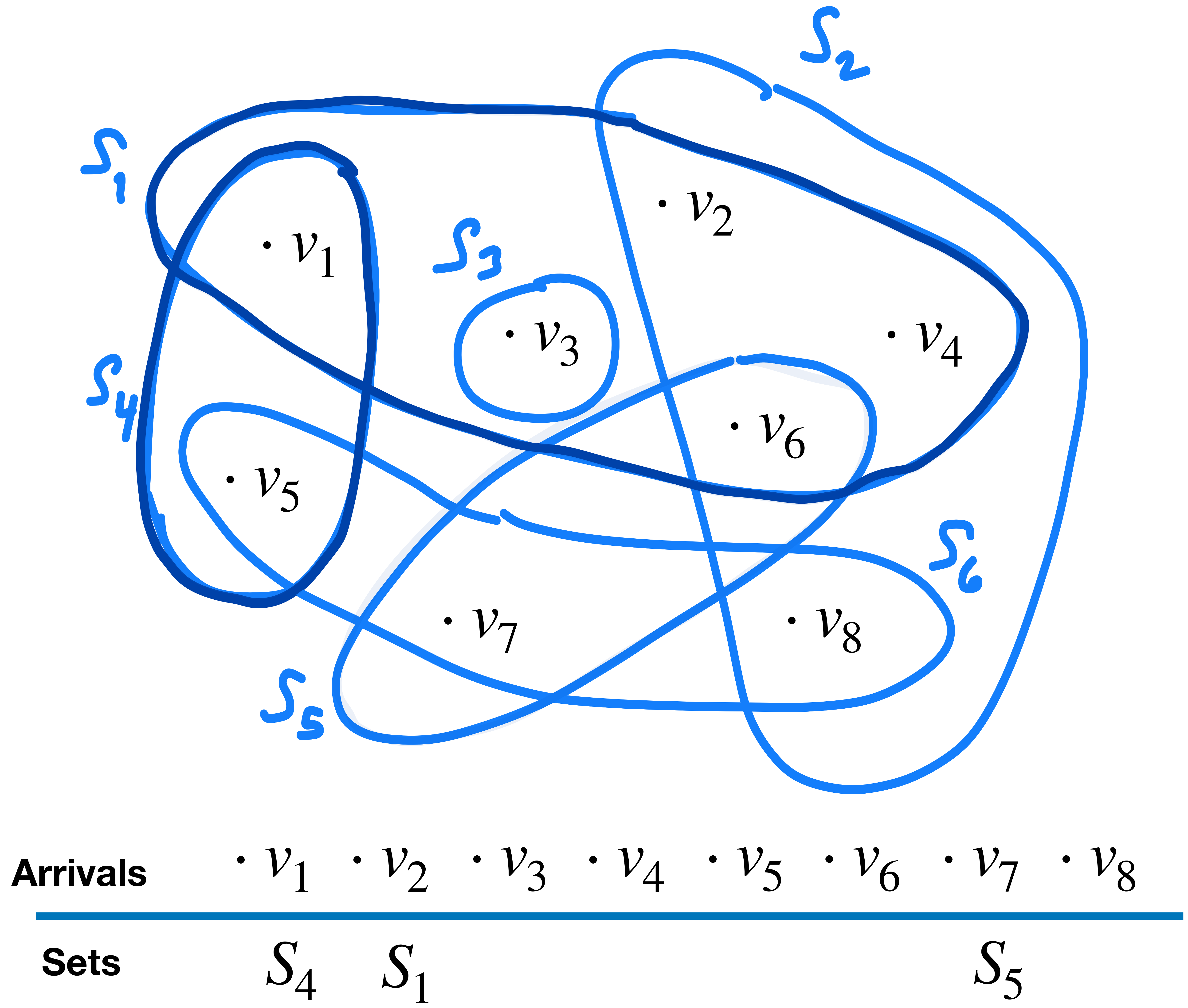
Online Set Cover



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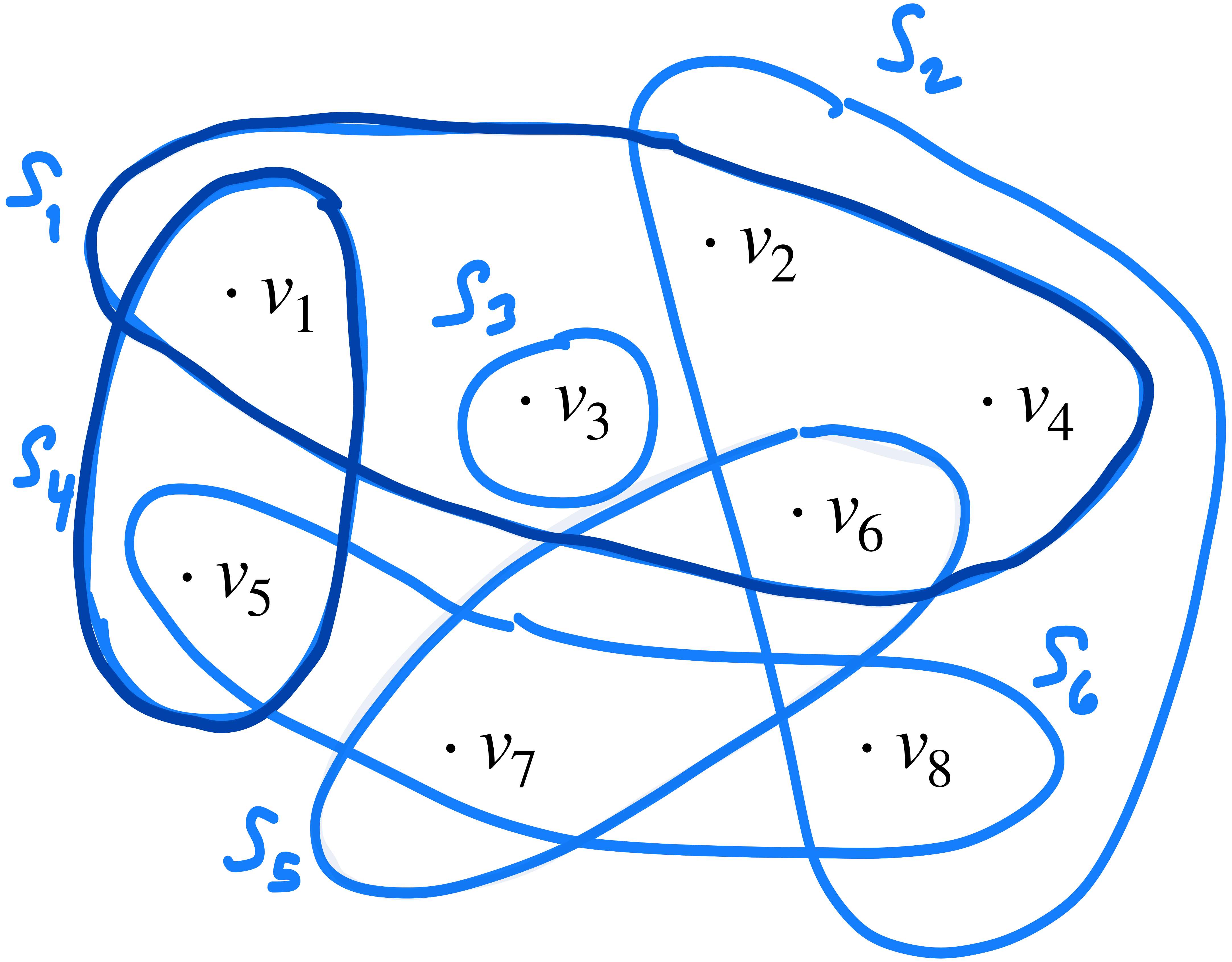

Arrivals $\cdot\, v_1$ $\cdot\, v_2$ $\cdot\, v_3$ $\cdot\, v_4$ $\cdot\, v_5$ $\cdot\, v_6$ $\cdot\, v_7$ $\cdot\, v_8$

Sets S_4 S_1 S_5 S_6

Online Set Cover:

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Goals:

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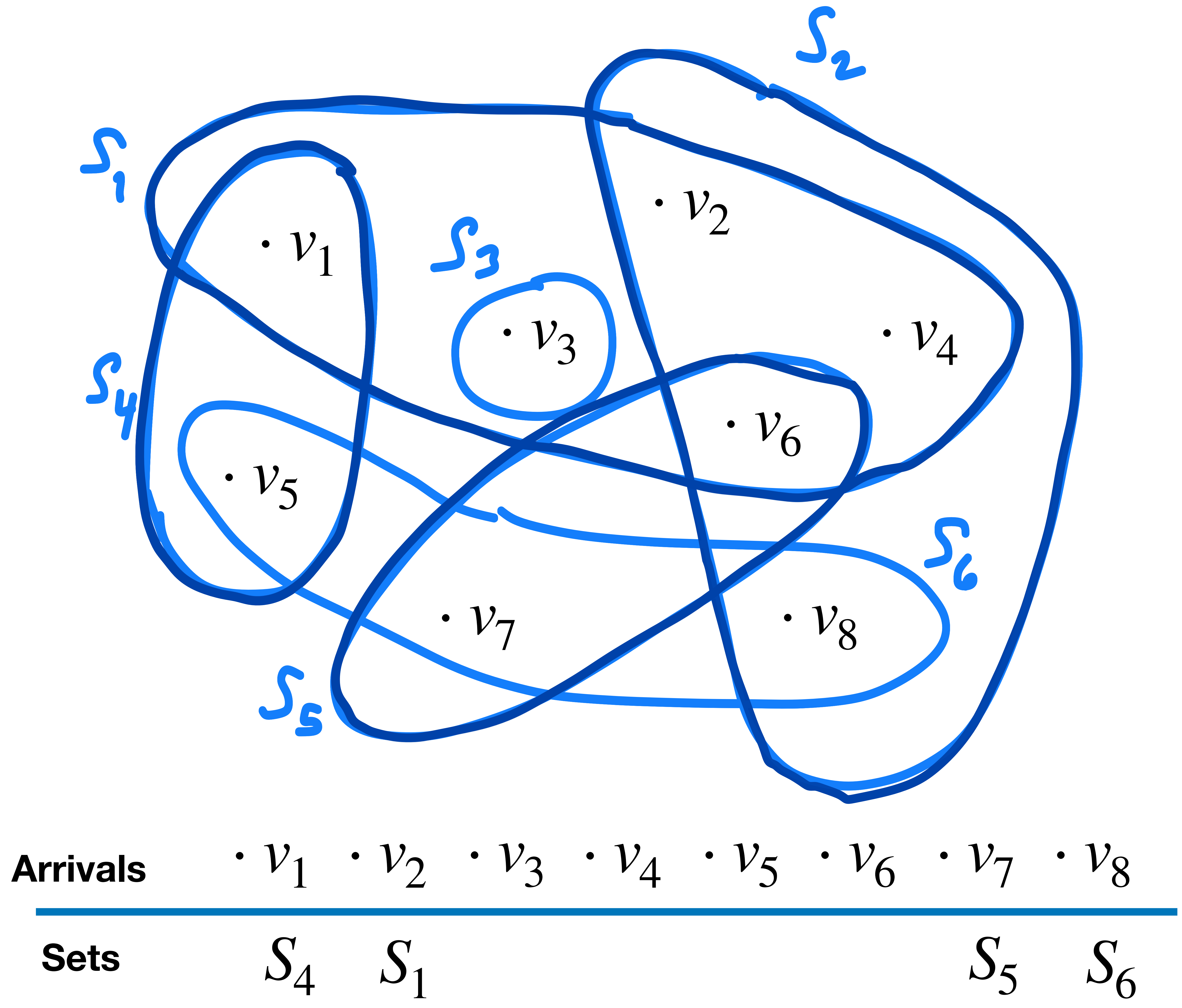

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Goals:

- Satisfy constraints online
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Prophet Set Cover

Prophet Covering

- n known constraint distributions $\mathscr{D}_i$
- Round i: draw $v_i \sim \mathscr{D}_i$, then satisfy it
- Goal: compete with $\mathbb{E}_{\mathscr{D}}[OPT(v_1, \ldots, v_n)]$

Prophet Set Cover

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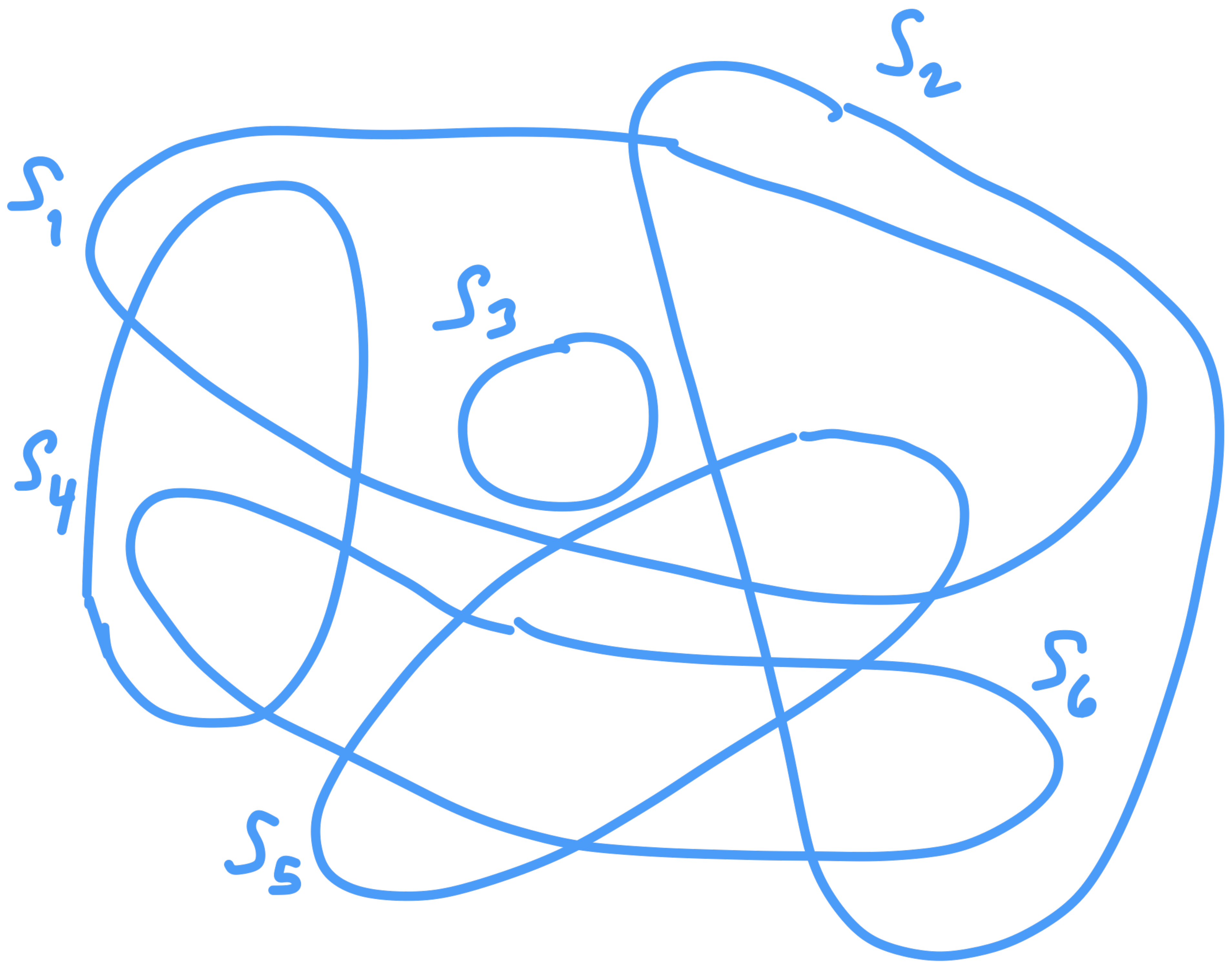

sampled constraints $v_i \sim \mathscr{D}_i$

Prophet Set Cover

Prophet Covering

- n known constraint distributions $\mathcal{D}_i$
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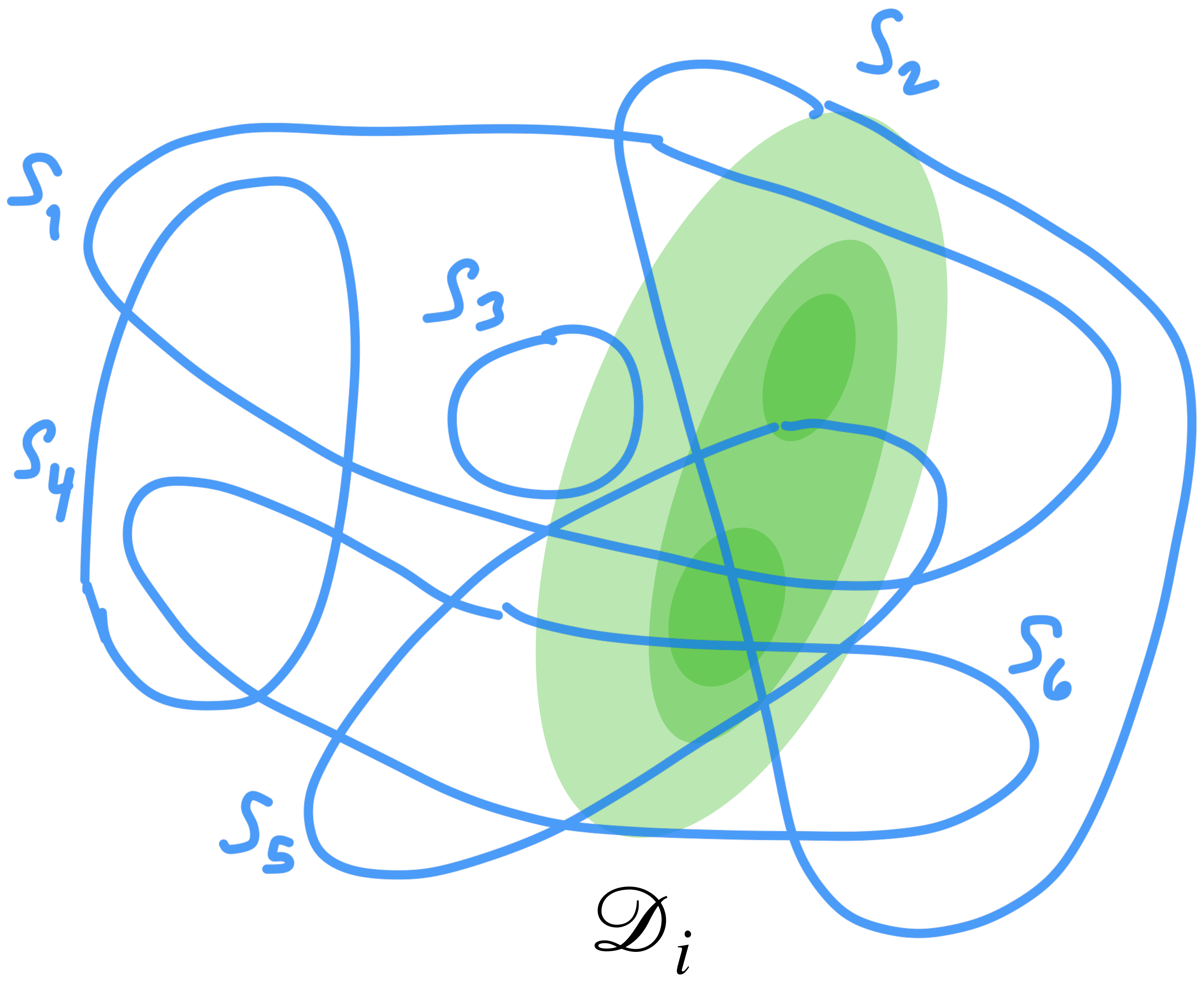

sampled constraints $v_i \sim \mathcal{D}_i$

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- n known constraint distributions $\mathscr{D}_i$
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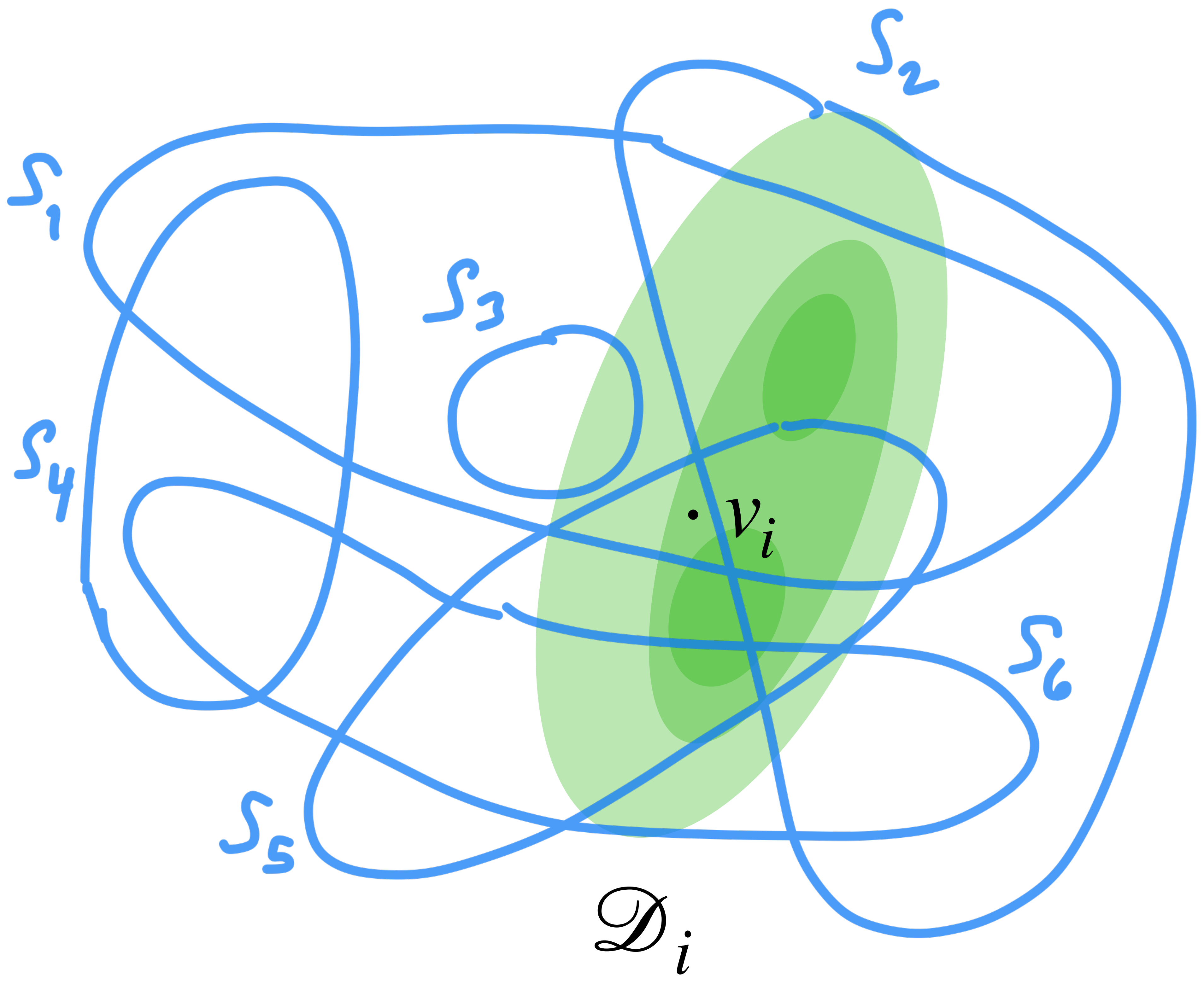

sampled constraints $v_i \sim \mathscr{D}_i$

The Landscape

Arrival Order \ Instance	Stochastic	Adversarial
RO		
Adversarial		

here $n =$ # elements

$m =$ # sets

$N =$ universe size

The Landscape

Arrival Order	Instance: Stochastic	Instance: Adversarial
RO		
Adversarial		$\Theta(\log m \; \log n)$ [Alon Awerbuch Azar Buchbinder Naor '03] [Korman '04]

here $n =$ # elements

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The Landscape

Instance

Arrival Order	Stochastic	Adversarial
RO		
Adversarial		$\Theta(\log m \; \log n)$ [Alon Awerbuch Azar Buchbinder Naor '03] [Korman '04]

What makes online set cover (online covering) harder than offline?

here $n =$ # elements

$m =$ # sets

$N =$ universe size

The Landscape

	Instance: Stochastic	Instance: Adversarial
Arrival Order: RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh ’08]	
Arrival Order: Adversarial		$\Theta(\log m \ \log n)$ [Alon Awerbuch Azar Buchbinder Naor ’03] [Korman ’04]

What makes online set cover (online covering) harder than offline?

here $n =$ # elements

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The Landscape

Instance

Arrival Order		Stochastic	Adversarial
	RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh '08] constraints i.i.d. from known $\mathscr{D}$	
	Adversarial		$\Theta(\log m \; \log n)$ [Alon Awerbuch Azar Buchbinder Naor '03] [Korman '04]

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The Landscape

Arrival Order \ Instance	Stochastic	Adversarial
RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh ’08] constraints i.i.d. from known $\mathscr{D}$	(secretary setting)
Adversarial		$\Theta(\log m \ \log n)$ [Alon Awerbuch Azar Buchbinder Naor ’03] [Korman ’04]

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here $n =$ # elements

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The Landscape

Instance

Arrival Order	Stochastic	Adversarial
RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh '08] constraints i.i.d. from known $\mathscr{D}$	$\Theta(\log mn)$ [Gupta K Levin '21] (secretary setting)
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The Landscape

Arrival Order \ Instance	Stochastic	Adversarial
RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh ’08] constraints i.i.d. from known $\mathscr{D}$	$\Theta(\log mn)$ [Gupta K Levin ’21] (secretary setting)
Adversarial	(prophet setting) constraints indep. from $\mathscr{D}_i$	$\Theta(\log m \ \log n)$ [Alon Awerbuch Azar Buchbinder Naor ’03] [Korman ’04]

What makes online set cover (online covering) harder than offline?

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The Landscape

Instance

Arrival Order		Stochastic	Adversarial
	RO	$O(\log mN)$ [Grandoni Gupta Leonardi Miettinen Sankowski Singh '08] constraints i.i.d. from known $\mathscr{D}$	$\Theta(\log mn)$ [Gupta K Levin '21] (secretary setting)
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What makes online set cover (online covering) harder than offline?

here $n =$ # elements

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The Landscape

Instance

Arrival Order	Stochastic	Adversarial
RO	$\Theta(\log mn)$ [this work] constraints i.i.d. from known $\mathscr{D}$	$\Theta(\log mn)$ [Gupta K Levin '21] (secretary setting)
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What makes online set cover (online covering) harder than offline?

here $n =$ # elements

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The Landscape

Instance

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(2-stage prophet)

What makes online set cover (online covering) harder than offline?

here $n =$ # elements

$m =$ # sets

$N =$ universe size

The Landscape

Arrival Order \ Instance	Stochastic	Adversarial
RO	$\Theta(\log mn)$ [this work] constraints i.i.d. from known $\mathscr{D}$	$\Theta(\log mn)$ [Gupta K Levin '21] (secretary setting)
Adversarial	$\Theta(\log mn)$ [this work] (prophet setting) constraints indep. from $\mathscr{D}_i$	$\Theta(\log m \ \log n)$ [Alon Awerbuch Azar Buchbinder Naor '03] [Korman '04]

(2-stage prophet) (with-a-sample)

What makes online set cover (online covering) harder than offline?

here $n =$ # elements

$m =$ # sets

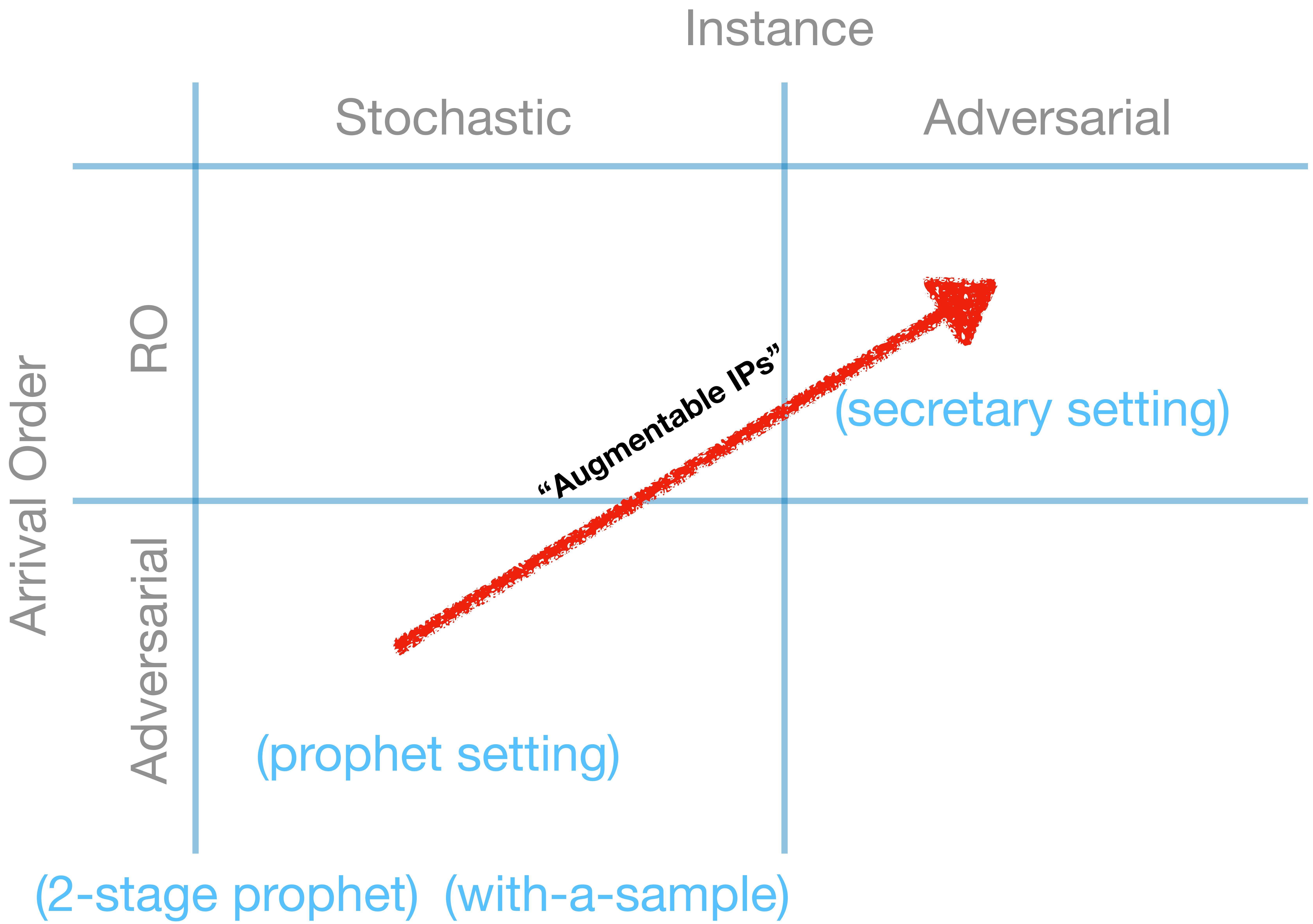
$N =$ universe size

Overview

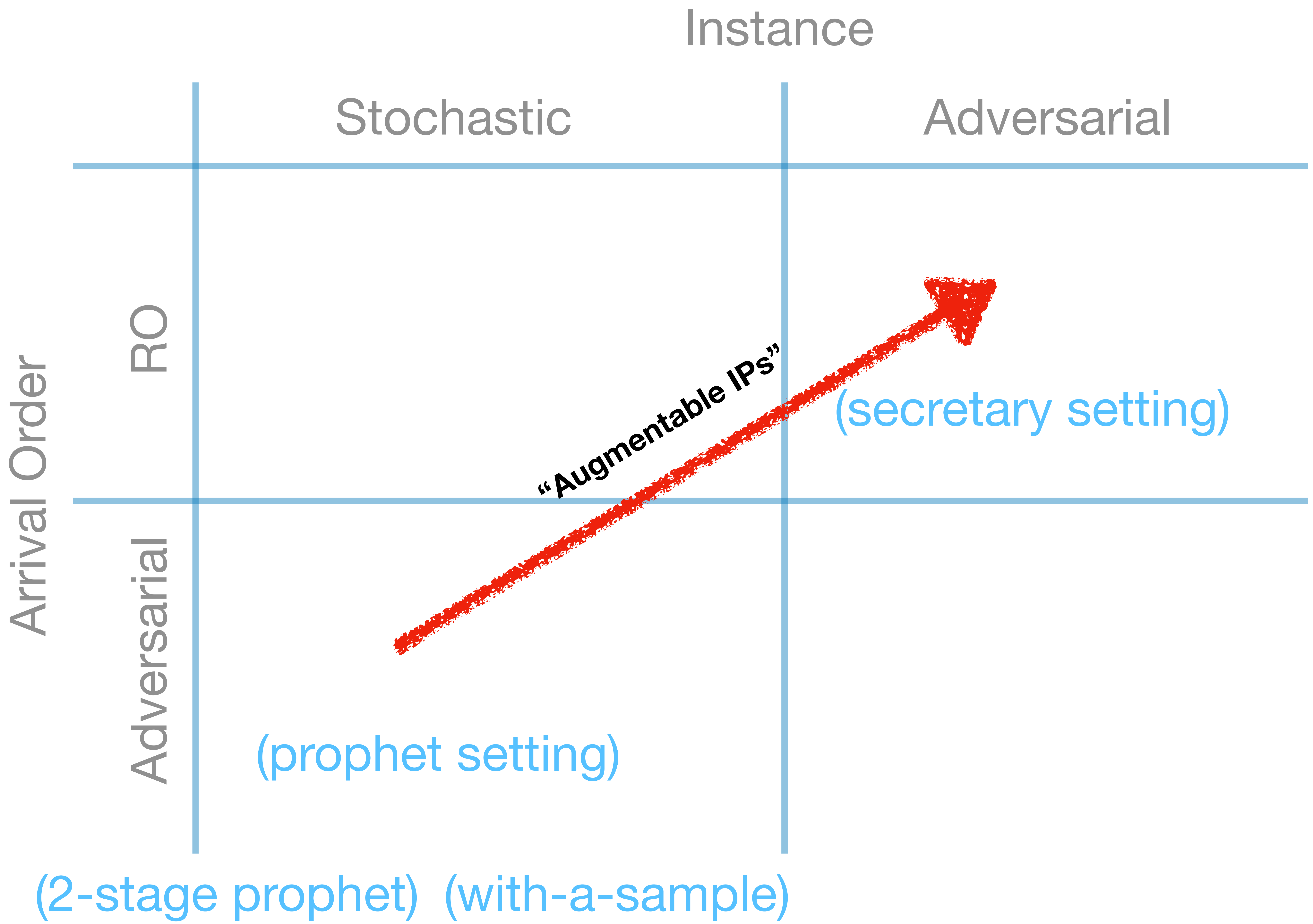
Arrival Order \ Instance	Stochastic	Adversarial
RO		(secretary setting)
Adversarial	(prophet setting)	

(2-stage prophet) (with-a-sample)

Overview

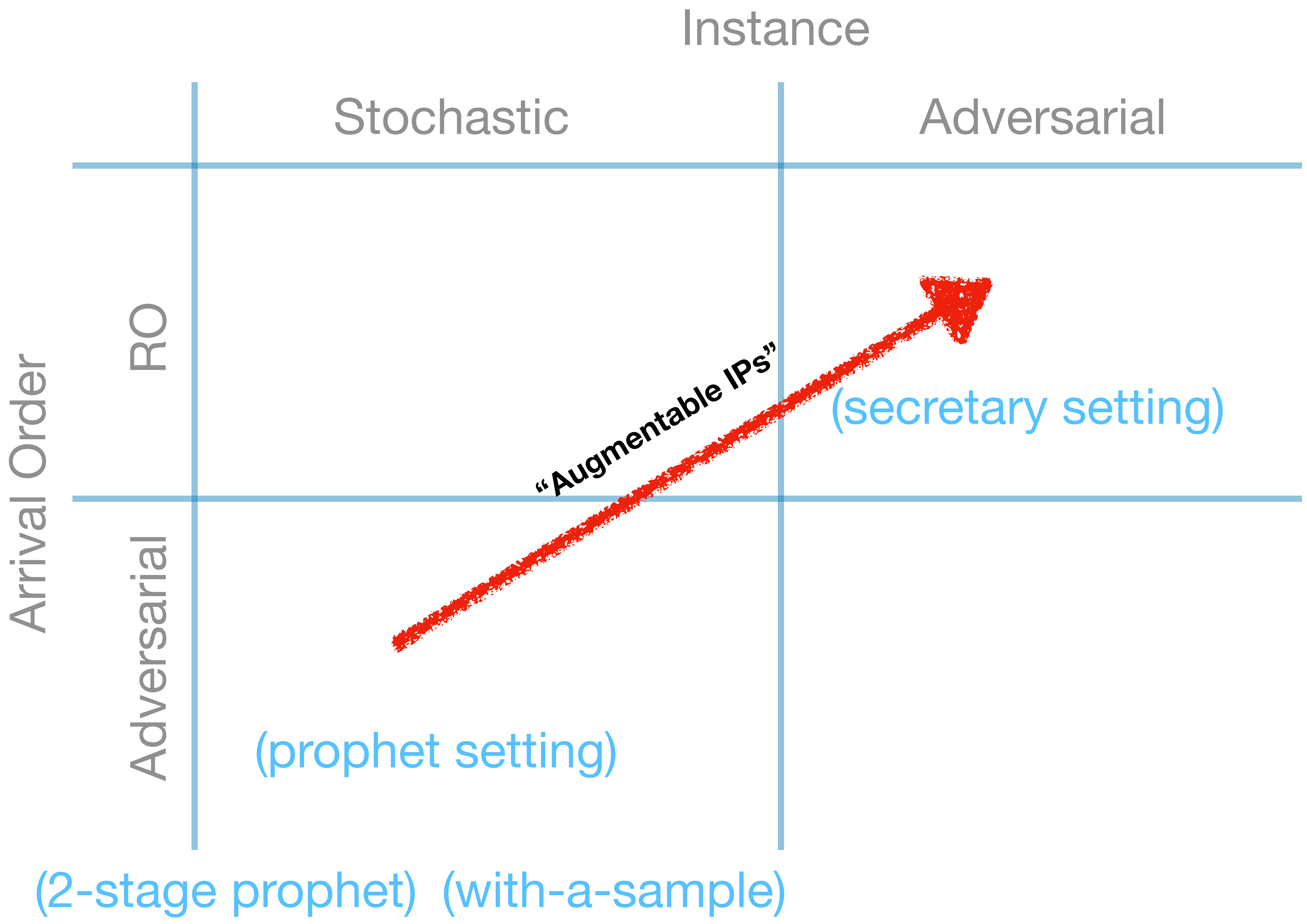

Instance
Stochastic
Adversarial
Arrival Order
RO
Adversarial
"Augmentable IPs"
(secretary setting)
(prophet setting)
(2-stage prophet)
(with-a-sample)

Overview



Augmentable IPs:

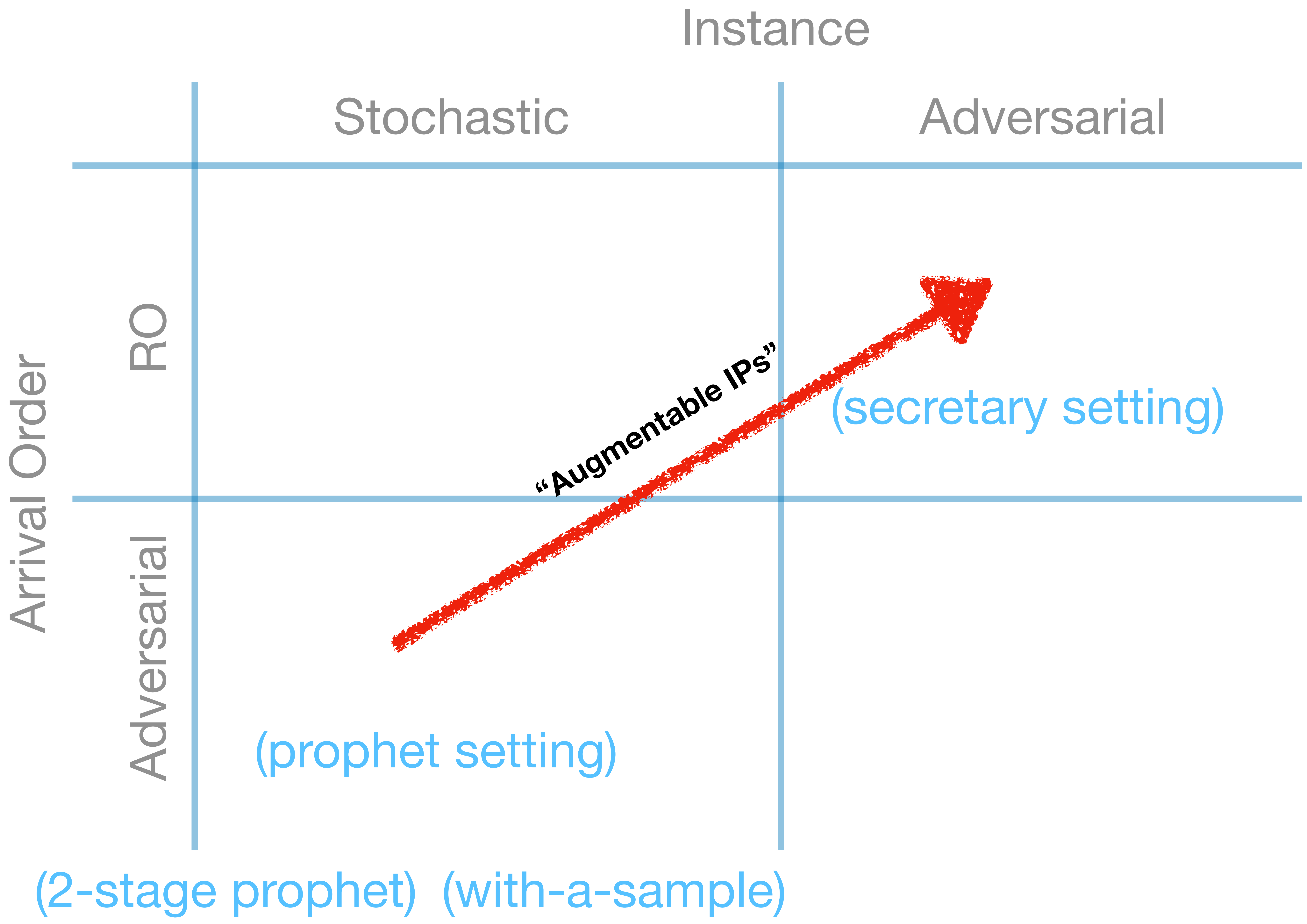
Overview



Augmentable IPs:

- Covering IPs + box constr.

Overview



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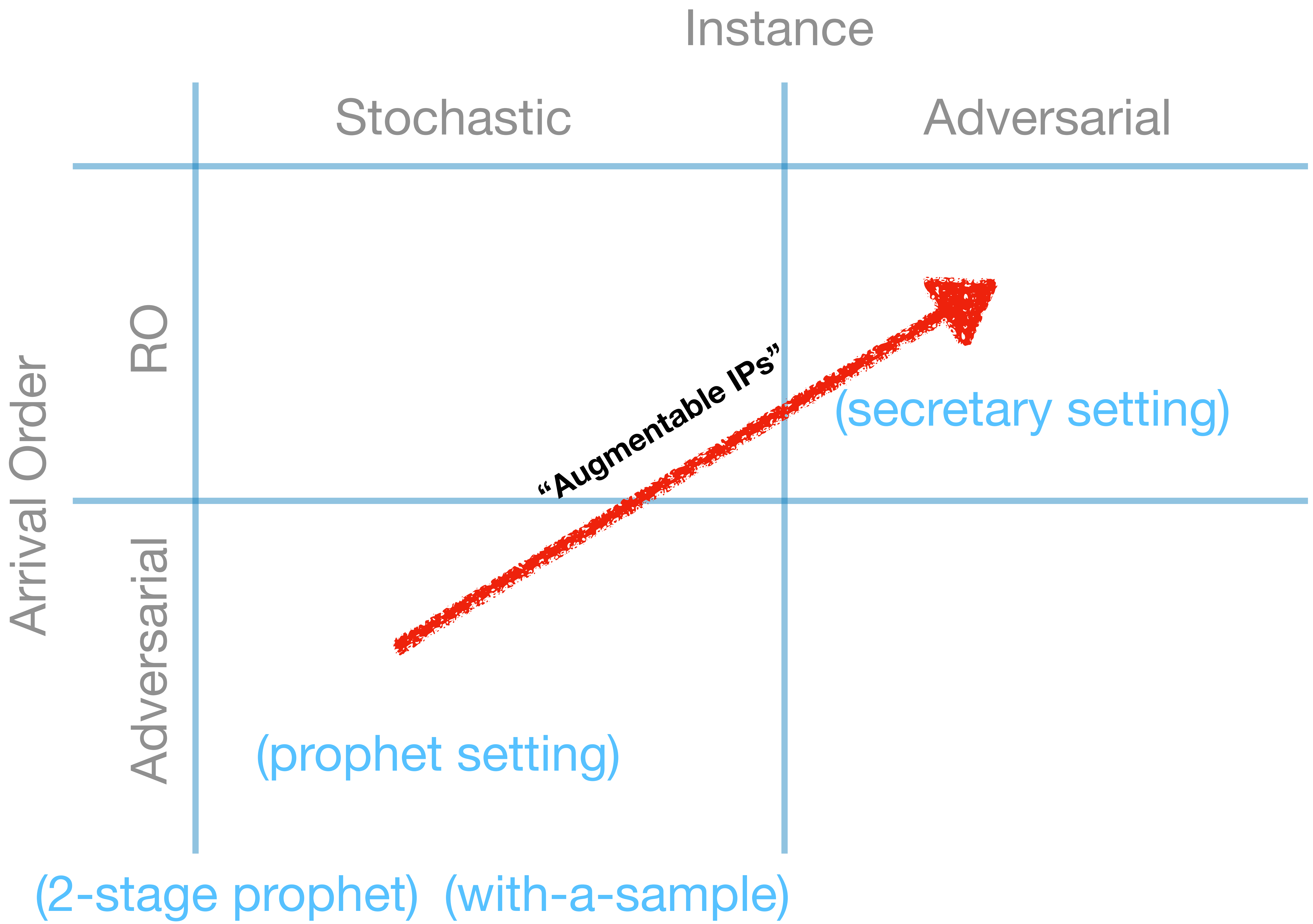
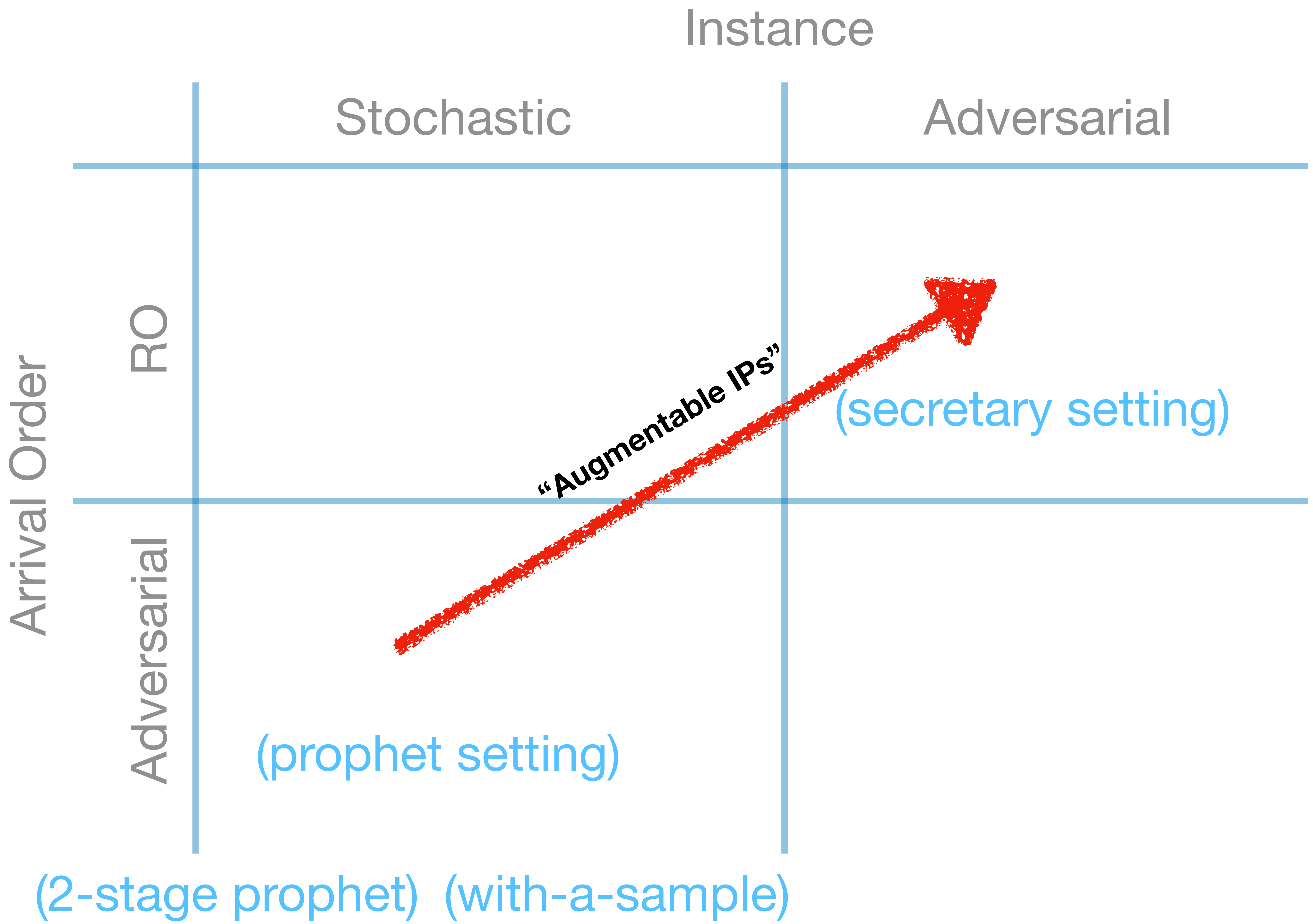
Overview



Augmentable IPs:

- Covering IPs + box constr.
- Covering IPs
- Set Cover

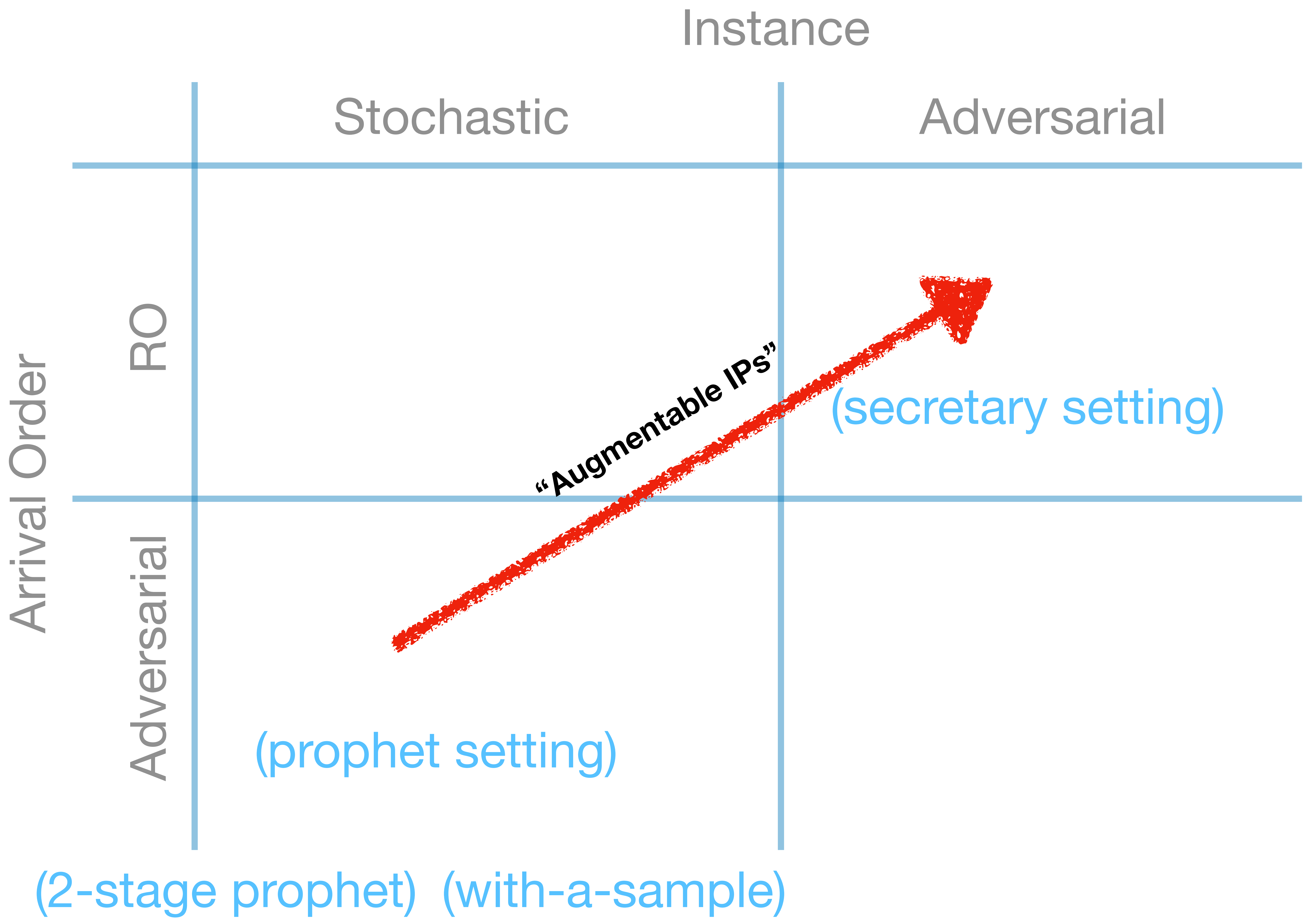
Overview



Augmentable IPs:

- Covering IPs + box constr.
- Covering IPs
- Set Cover
- Set Multicover

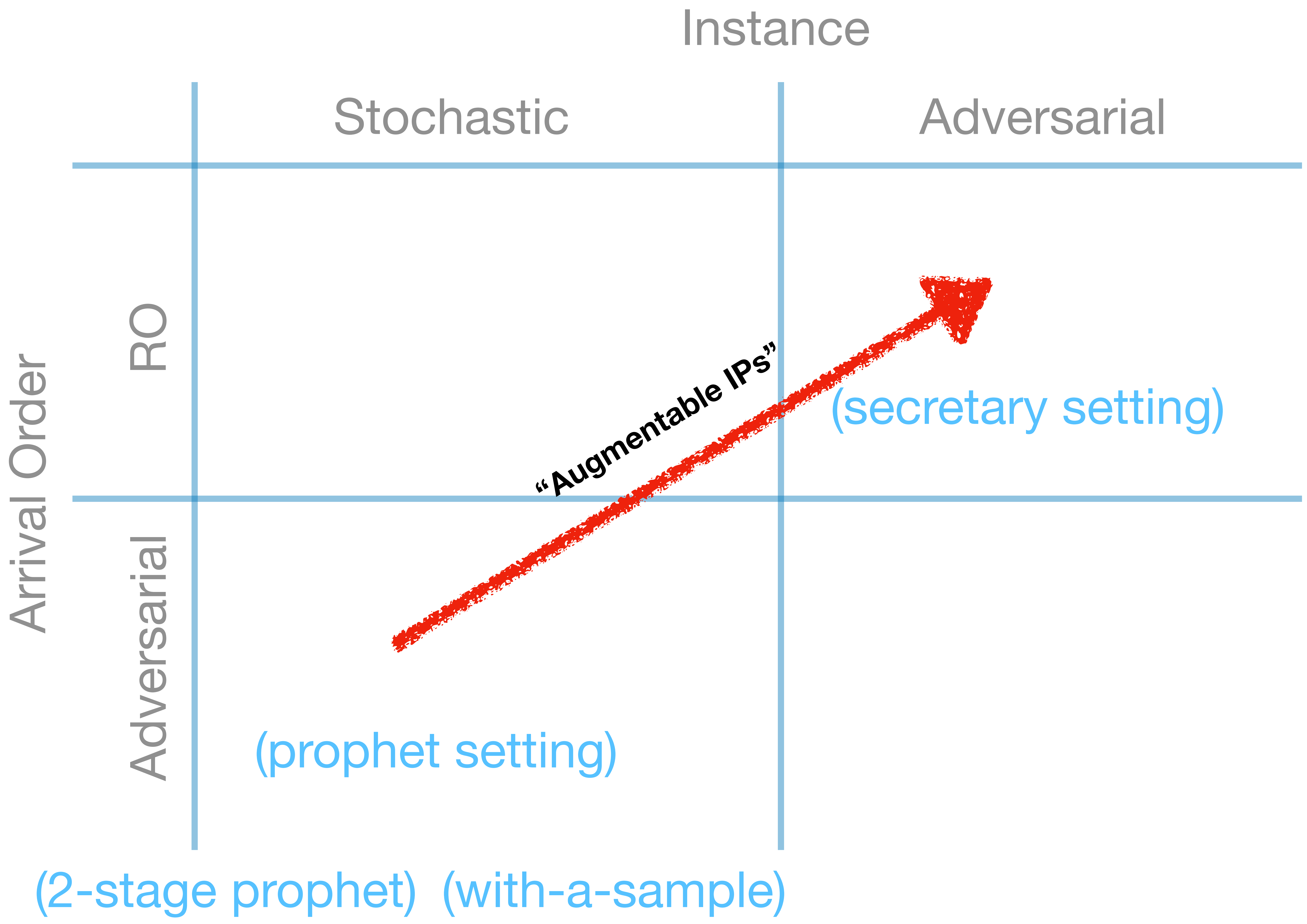
Overview



Augmentable IPs:

- Covering IPs + box constr.
- Covering IPs
- Set Cover
- Set Multicover
- Metric Facility Location

Overview



Augmentable IPs:

- Covering IPs + box constr.
- Covering IPs
- Set Cover
- Set Multicover
- Metric Facility Location
- Nonmetric Facility Location

Beyond-Worst-Case Models

Beyond-Worst-Case Models

Beyond-Worst-Case Models

Prophet

- n known constraint distributions $\mathcal{D}_i$
- Round i: draw $v_i \sim \mathcal{D}_i$, then satisfy it
- Goal: compete with $\mathbb{E}_{\mathcal{D}}[OPT(v_1, \ldots, v_n)]$

Beyond-Worst-Case Models

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2-Stage Prophet

- n known constraint distributions $\mathscr{D}_i$
- Beforehand: buy partial solution
- Round i: draw $v_i \sim \mathscr{D}_i$, satisfy at cost $\times \lambda$
- Goal: compete with 2-stage online OPT

Beyond-Worst-Case Models

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With-a-Sample

- Adversarial arrivals $(v_1, \ldots, v_i, \ldots, v_n)$
- Beforehand: observe each v_i w.p. $\alpha \in [0,1]$
- Round i: see v_i, then satisfy it

Beyond-Worst-Case Models

Prophet *(Single Sample)*

- Observe a sample $\hat{v}_i \sim \mathcal{D}_i$ from each $\mathcal{D}_i$
- Round i: draw $v_i \sim \mathcal{D}_i$, then satisfy it
- Goal: compete with $\mathbb{E}_{\mathcal{D}}[OPT(v_1, \ldots, v_n)]$

2-Stage Prophet *(from Samples)*

- Observe λ samples $\hat{v}_i \sim \mathcal{D}_i$ from each $\mathcal{D}_i$
- Beforehand: buy partial solution
- Round i: draw $v_i \sim \mathcal{D}_i$, satisfy at cost $\times \lambda$
- Goal: compete with 2-stage online OPT

With-a-Sample

- Adversarial arrivals $(v_1, \ldots, v_i, \ldots, v_n)$
- Beforehand: observe each v_i w.p. $\alpha \in [0,1]$
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Prophet Set Cover (single sample)

Set cover: (single sample prophet)

- n unknown $\mathscr{D}_1, \ldots, \mathscr{D}_n$
- Observe n samples $\hat{v}_i \sim \mathscr{D}_i$
- Round i: draw constraint $v_i \sim \mathscr{D}_i$

  and buy a set to satisfy it
- Goal: compete with $\mathbb{E}_{\mathscr{D}}[OPT(v_1, \ldots, v_n)]$

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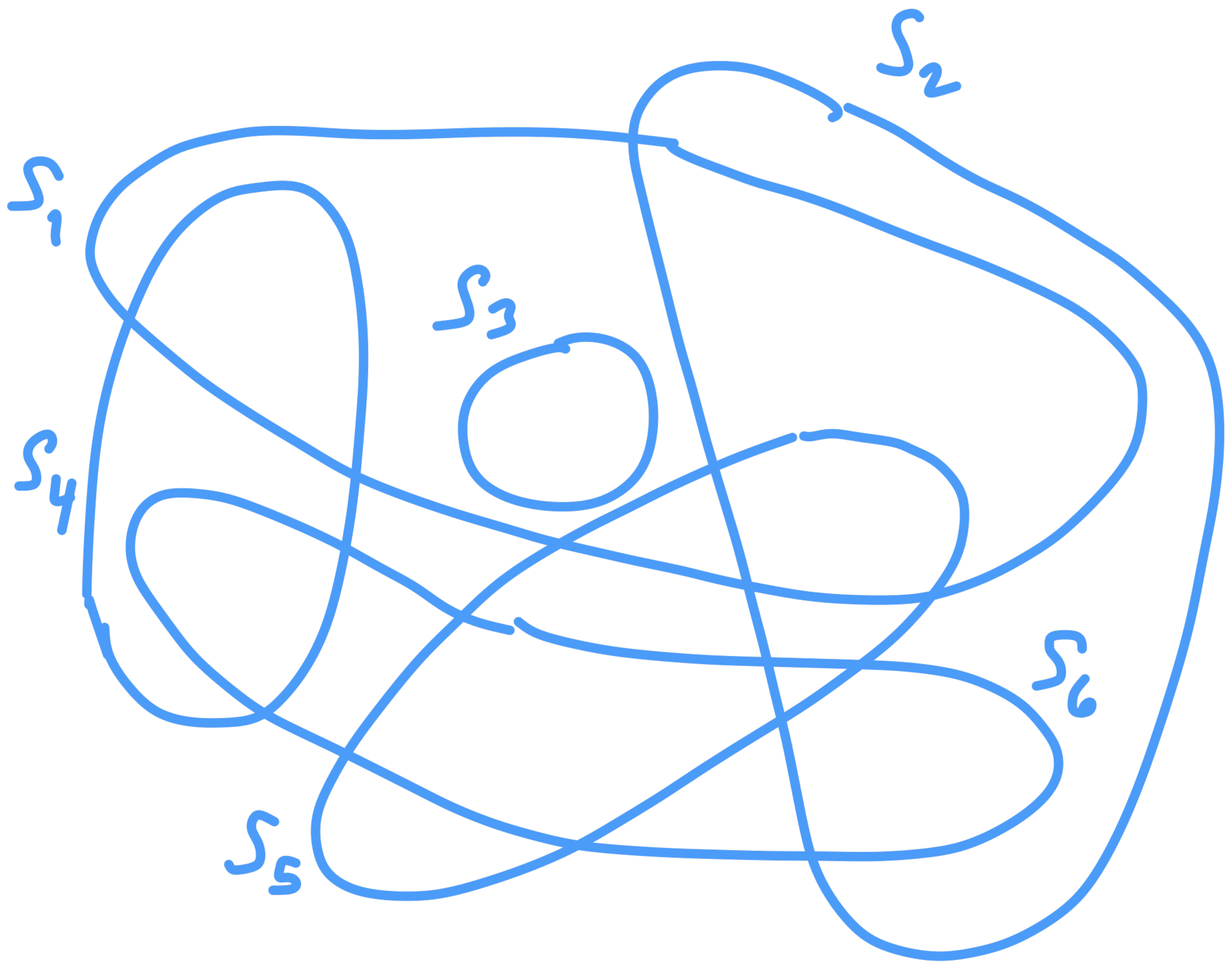

sample and constraints $\hat{v}_i, v_i \sim \mathscr{D}_i$

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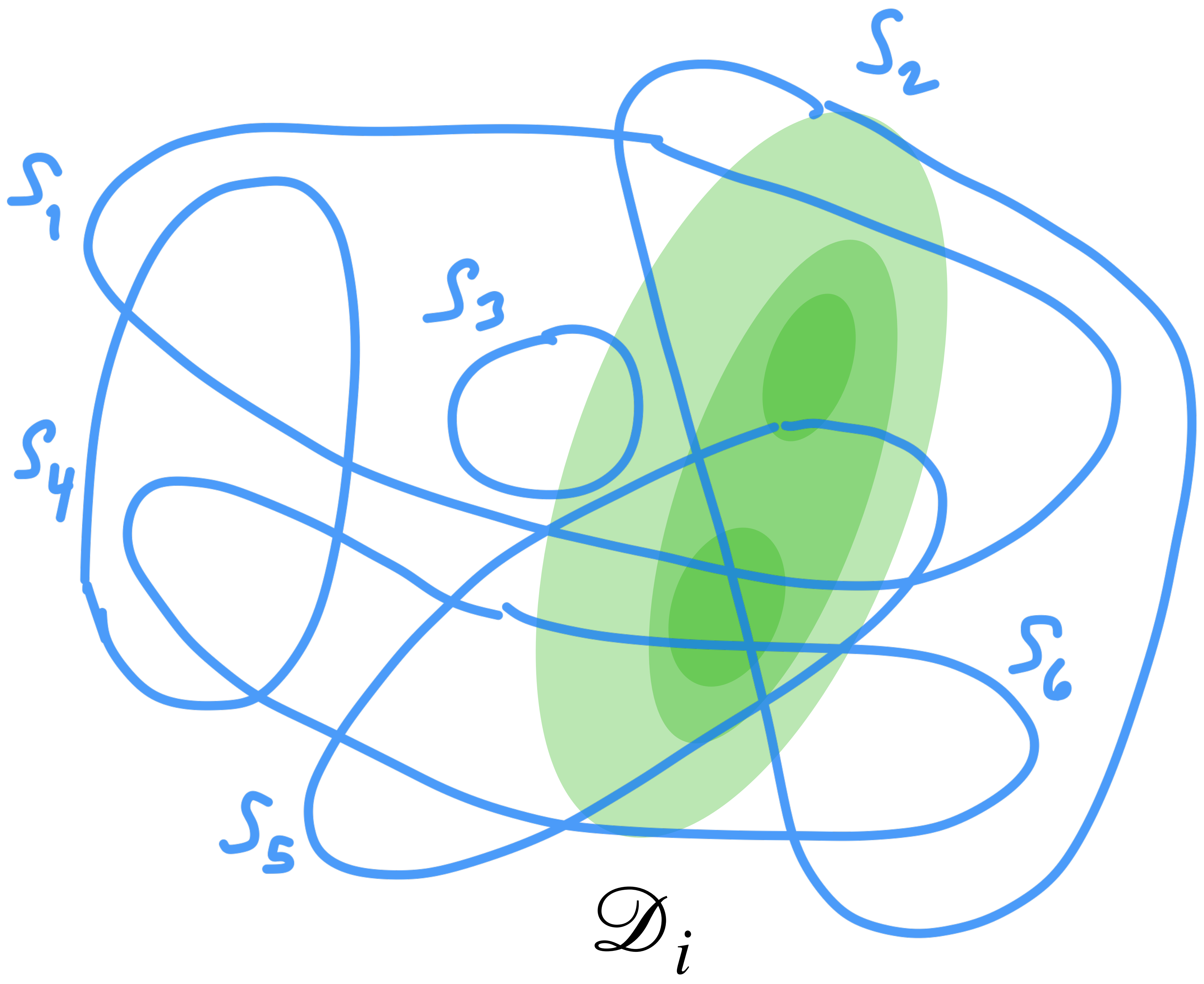

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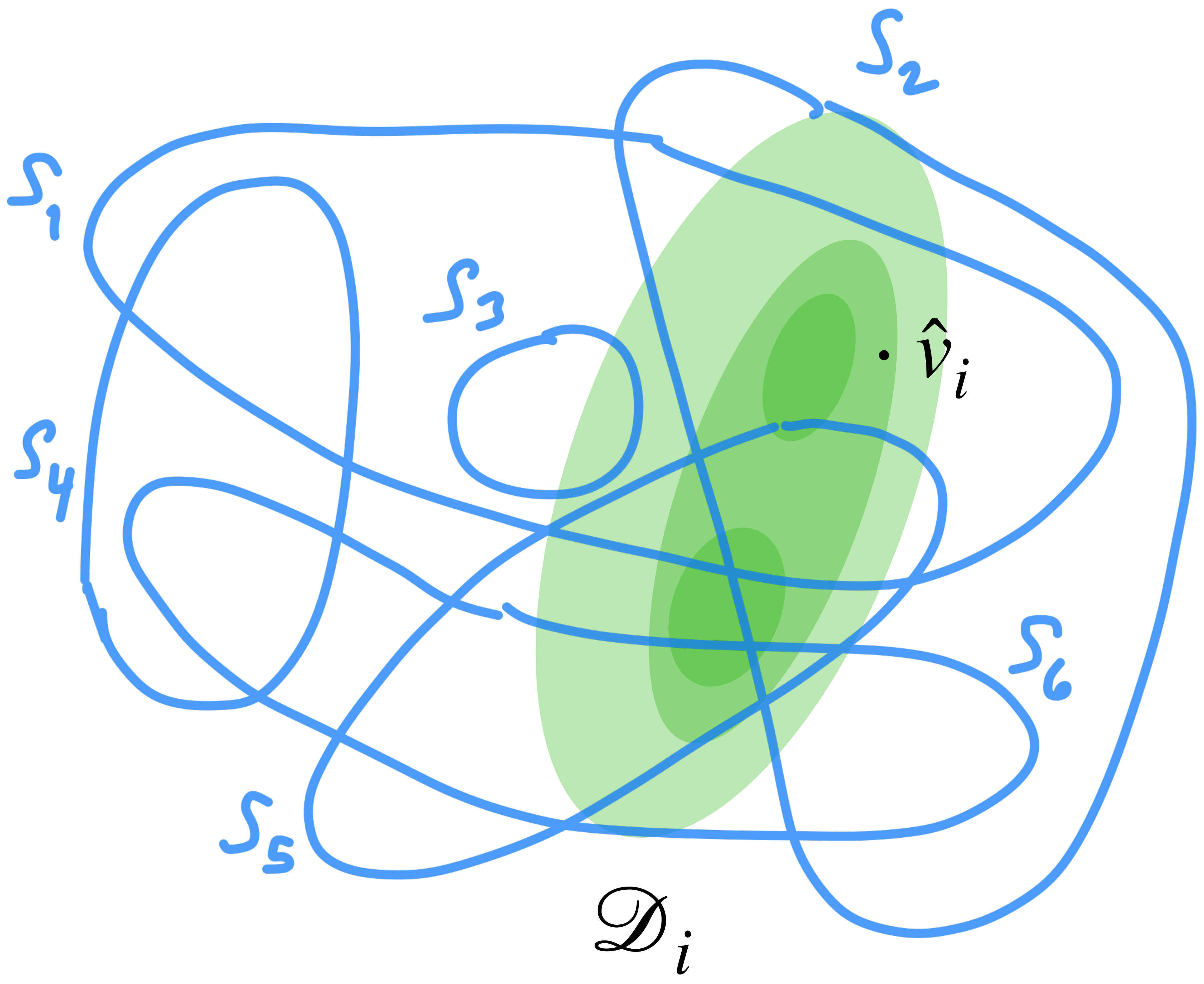

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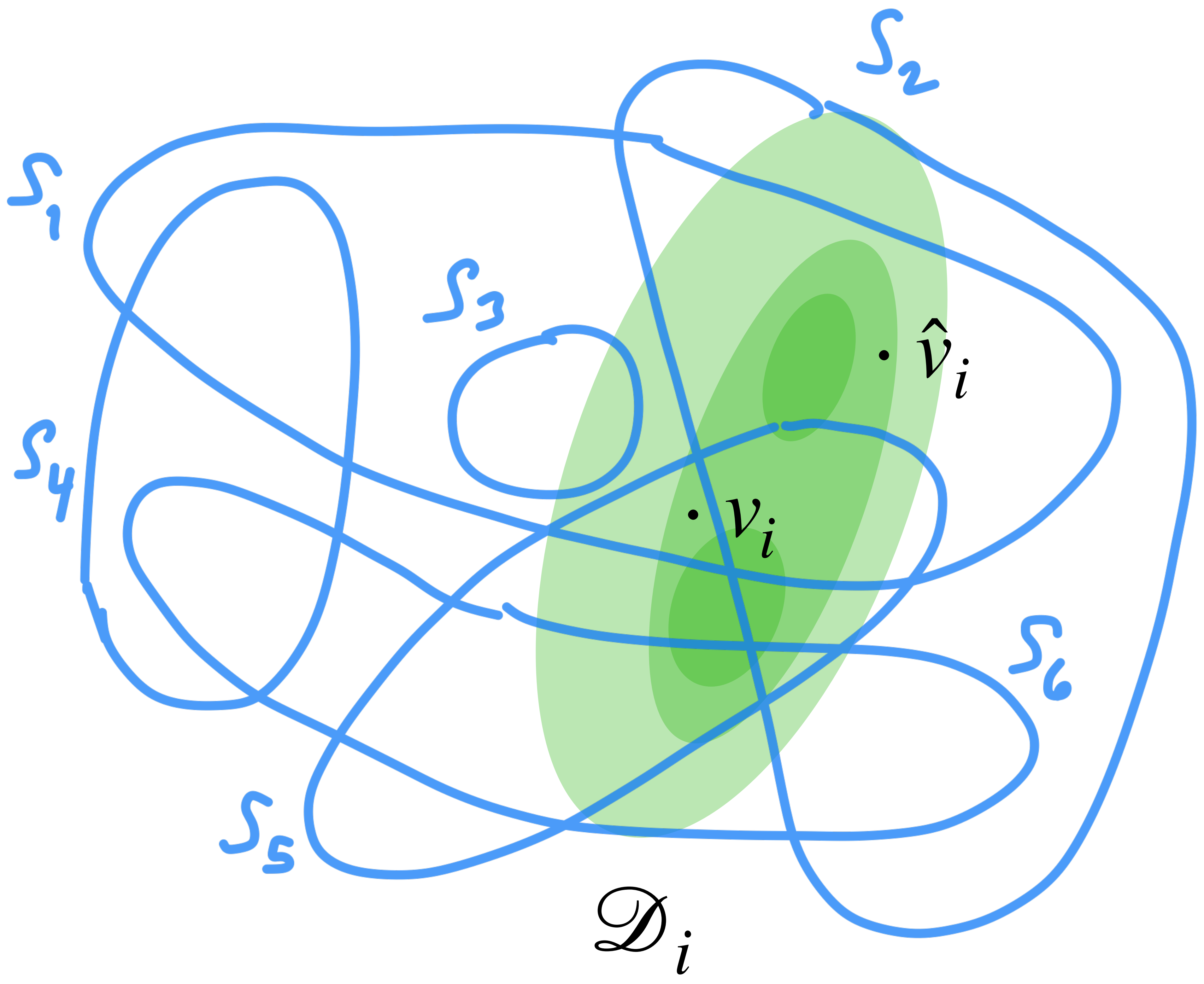

sample and constraints $\hat{v}_i, v_i \sim \mathscr{D}_i$

Single Sample Prophet Set Cover

SSP Set Cover

given samples $\hat{v}_i \sim \mathscr{D}_i$

run ROSC algo on $\hat{v}_1, \ldots, \hat{v}_n$

$\mathscr{C}$ = sets ROSC algo buys

for v_i arriving uncovered (round t):

if $v_i \in C$ for some $C \in \mathscr{C}$:

buy this C

else:

(Backup)

buy arbitrary $S \ni v_i$

Theorem (Gupta K Levin): This reduction to ROSC is an $O(\log mn)$-approximation for SSP Set Cover.

Single Sample Prophet Set Cover

SSP Set Cover

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Proof:

$$\mathbb{E}[c(\mathscr{C})] = \mathbb{E}[c(\text{LoC }(v_1, \ldots, v_n))]$$
$$\leq O(\log mn) \cdot \mathbb{E}[OPT] \qquad \text{by [GKL21]}$$

v_i backup costs satisfy

$$\mathbb{E}[c(v_i \text{ backup})] \leq \mathbb{E}[c(\text{LoC } \hat{v}_i \text{ backup})]$$
$$\mathbb{E}[c(v_i \text{ backups})] \leq \mathbb{E}[c(\text{LoC }(v_1, \ldots, v_n))] \,.$$

Single Sample Prophet Set Cover

SSP Set Cover

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$$\mathbb{E}[c(v_i \text{ backup})] \leq \mathbb{E}[c(\text{LoC } \hat{v}_i \text{ backup})]$$

why?

$$\implies \mathbb{E}[c(v_i \text{ backups})] \leq \mathbb{E}[c(\text{LoC }(v_1, \ldots, v_n))].$$

Single Sample Prophet Set Cover

ROSC algo (Learn or Cover)

(estimate $k = |OPT|$)

initialize $x \leftarrow \mathrm{k}/m$

for $\hat{v}_i$ arriving uncovered (round t)

(Cover)

buy random $S \sim x$

(Learn)

if $x_{\hat{v}} \leq (e-1)^{-1}$:

$x_S \leftarrow e \cdot x_S$ for all $S \ni \hat{v}_i$

$$x \leftarrow k\frac{x}{\|x\|}$$

(Backup)

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RO samples

Adv. seq.

Single Sample Prophet Set Cover

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RO samples $\hat{v}_3$ $\hat{S}_3$

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Adv. seq.

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RO samples $\hat{v}_1$ $\hat{S}_1$ $\hat{v}_2$ $\hat{v}_3$ $\hat{S}_3$ $\cdots$ $\hat{v}_i$ $\hat{S}_i$ $\hat{v}_n$ $\hat{S}_n$

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RO samples $\hat{v}_1$ $\hat{S}_1$ $\hat{v}_2$ $\hat{v}_3$ $\hat{S}_3$ $\cdots$ $\hat{v}_i$ $\hat{S}_i$ $\cdots$ $\hat{v}_n$ $\hat{S}_n$

Adv. seq.

Single Sample Prophet Set Cover

ROSC algo (Learn or Cover)

(estimate $k = |OPT|$)

initialize $x \leftarrow$ k$/m$

for $\hat{v}_i$ arriving uncovered (round t)

(Cover)

buy random $S \sim x$

(Learn)

if $x_{\hat{v}} \leq (e-1)^{-1}$:

$x_S \leftarrow e \cdot x_S$ for all $S \ni \hat{v}_i$

$x \leftarrow k \dfrac{x}{\|x\|}$

(Backup)

buy arbitrary $S \ni \hat{v}_i$

buy arbitrary $S \ni v_i$

<u>Theorem</u> (Gupta K Levin): This reduction to ROSC is an $O(\log mn)$-approximation for SSP Set Cover.

$\mathbb{E}[c(v_i$ backup$)] \leq \mathbb{E}[c($LoC $\hat{v}_i$ backup$)]$ ← why?

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Adv. seq. v_1 S_1

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Adv. seq.	$v_1\ S_1$	$v_2\ S_2$					

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Single Sample Prophet Set Cover

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Adv. seq.	v_1 S_1	v_2 S_2	v_3	$\cdots$			

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Adv. seq.	v_1 S_1	v_2 S_2	v_3	$\cdots$	v_i S_i	$\cdots$	

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Adv. seq.	v_1 S_1	v_2 S_2	v_3	$\cdots$	v_i S_i	$\cdots$	v_n S_n

For Set Cover,

competitive ratio guarantee:

Prophet *(Single Sample)*

2-Stage Prophet *(λ Samples)*

Adversarial With-a-Sample

For Set Cover,

competitive ratio guarantee:

Prophet *(Single Sample)*

$$2 \cdot \mathbb{E}[RO] = O(\log mn) \cdot OPT_{LP}$$

2-Stage Prophet *(λ Samples)*

Adversarial With-a-Sample

For Set Cover,

competitive ratio guarantee:

Prophet *(Single Sample)*

$$2 \cdot \mathbb{E}[RO] = O(\log mn) \cdot OPT_{LP}$$

2-Stage Prophet *(λ Samples)*

$$2 \cdot \mathbb{E}[RO] = O(\log mn) \cdot OPT_{Online}$$

Adversarial With-a-Sample

For Set Cover,

competitive ratio guarantee:

Prophet *(Single Sample)* — $2 \cdot \mathbb{E}[RO] = O(\log mn) \cdot OPT_{LP}$

2-Stage Prophet *(λ Samples)* — $2 \cdot \mathbb{E}[RO] = O(\log mn) \cdot OPT_{Online}$

Adversarial With-a-Sample — $\alpha^{-1} \cdot \mathbb{E}[RO] = O(\alpha^{-1} \log mn) \cdot OPT_{LP}$

More Generally,

RO algos for

Augmentable IPs:

- Covering IPs + box constr.
- Covering IPs
- Set Cover
- Set Multicover
- Metric Facility Location
- Nonmetric Facility Location

⇛

Prophet *(Single Sample)*

2-Stage Prophet *(from Samples)*

Adversarial With-a-Sample

More Generally,

RO algos for

Augmentable IPs:

- Covering IPs + box constr.

[GKL21] • Covering IPs

[GKL21] • Set Cover

this work • Set Multicover

[Mey01] • Metric Facility Location

this work • Nonmetric Facility Location

⇒

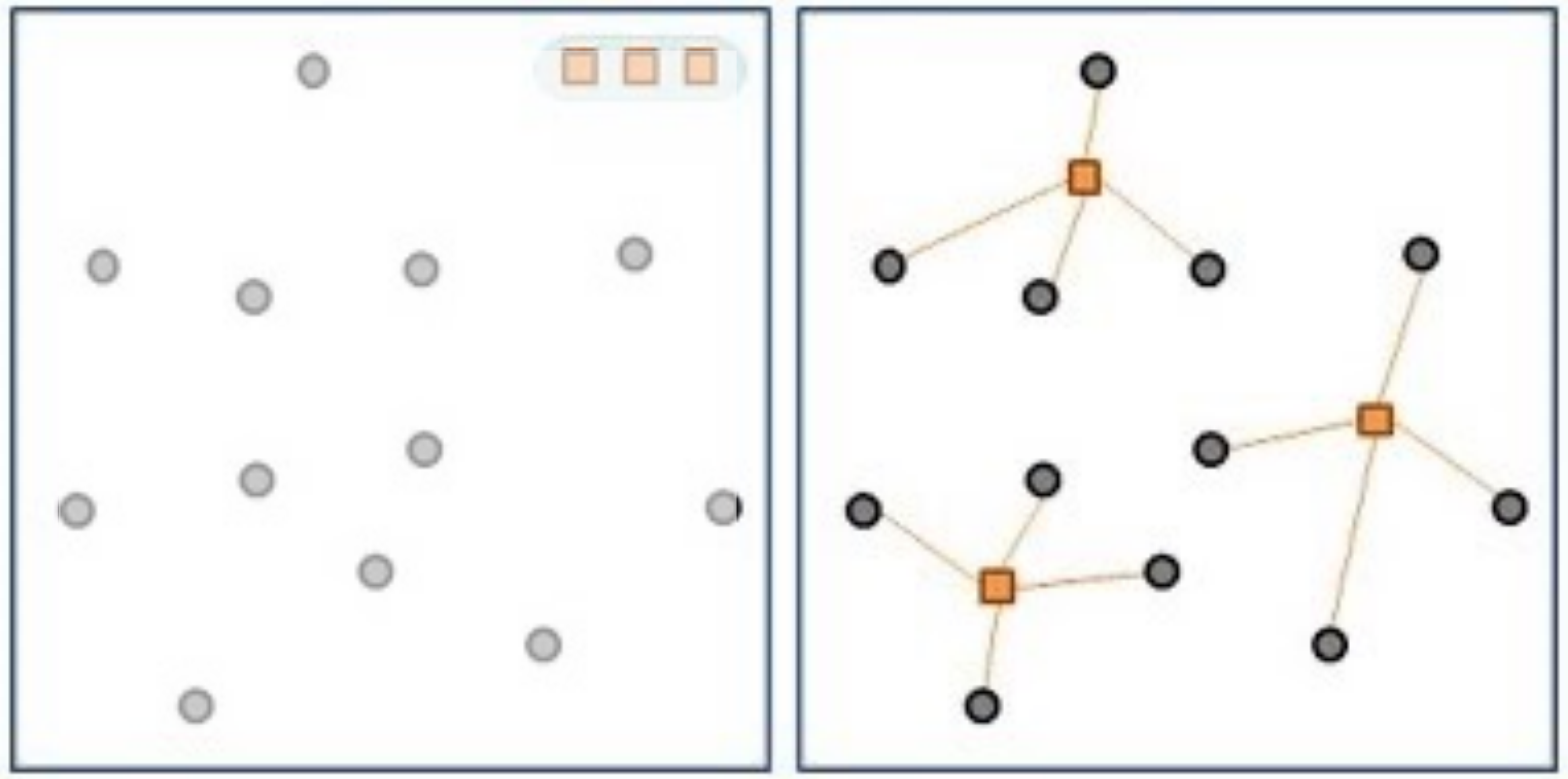
Prophet *(Single Sample)*

2-Stage Prophet *(from Samples)*

Adversarial With-a-Sample

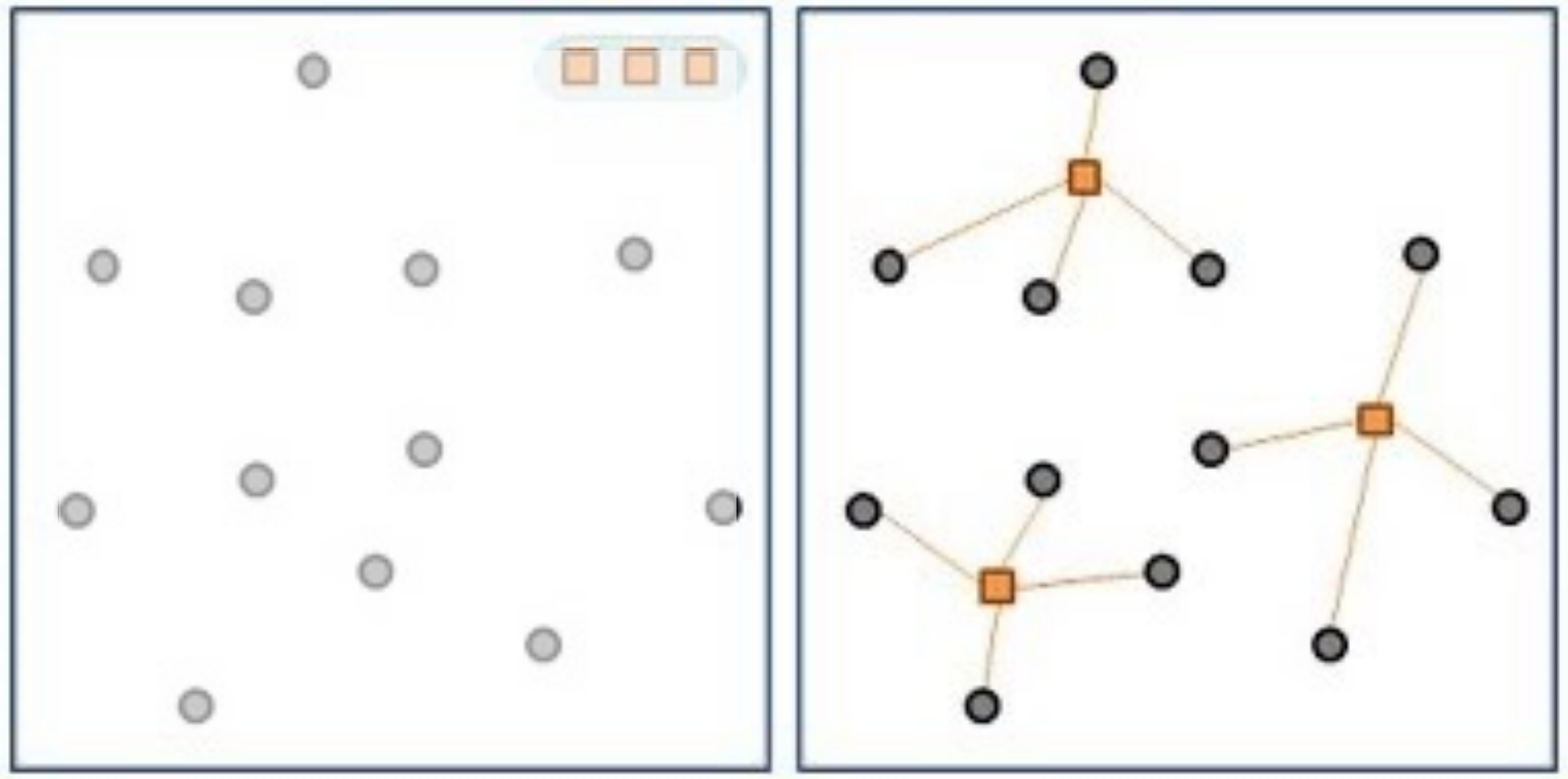
Example: Facility Location

Example: Facility Location



Example: Facility Location

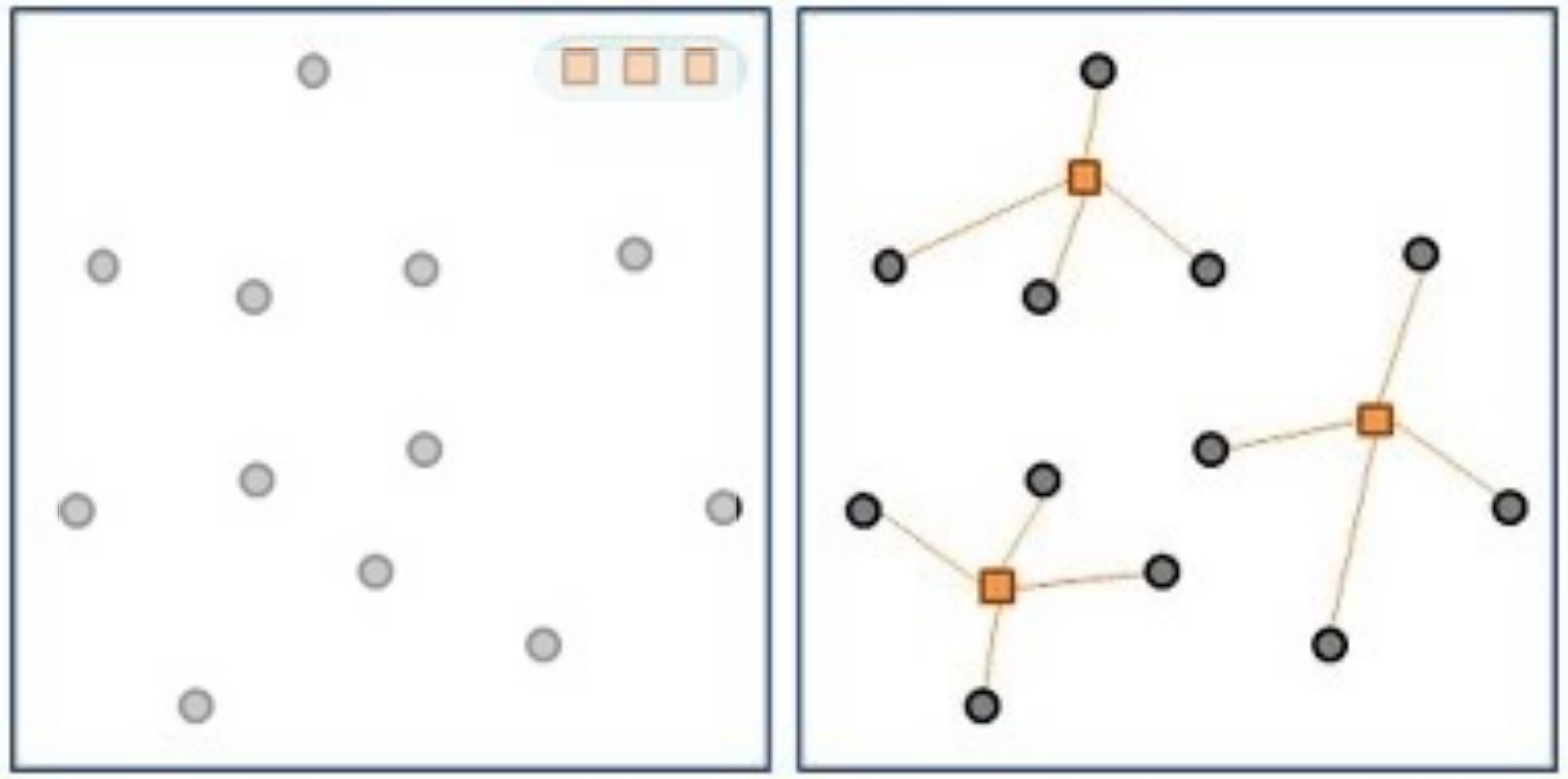
Random Order:



Example: Facility Location

Random Order:

- 8-approx [Meyerson '01]: open arriving i w.p. $\dfrac{c(\text{connect } i)}{c(\text{open } i)}$



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- 8-approx [Meyerson '01]: open arriving i w.p. $\dfrac{c(\text{connect } i)}{c(\text{open } i)}$
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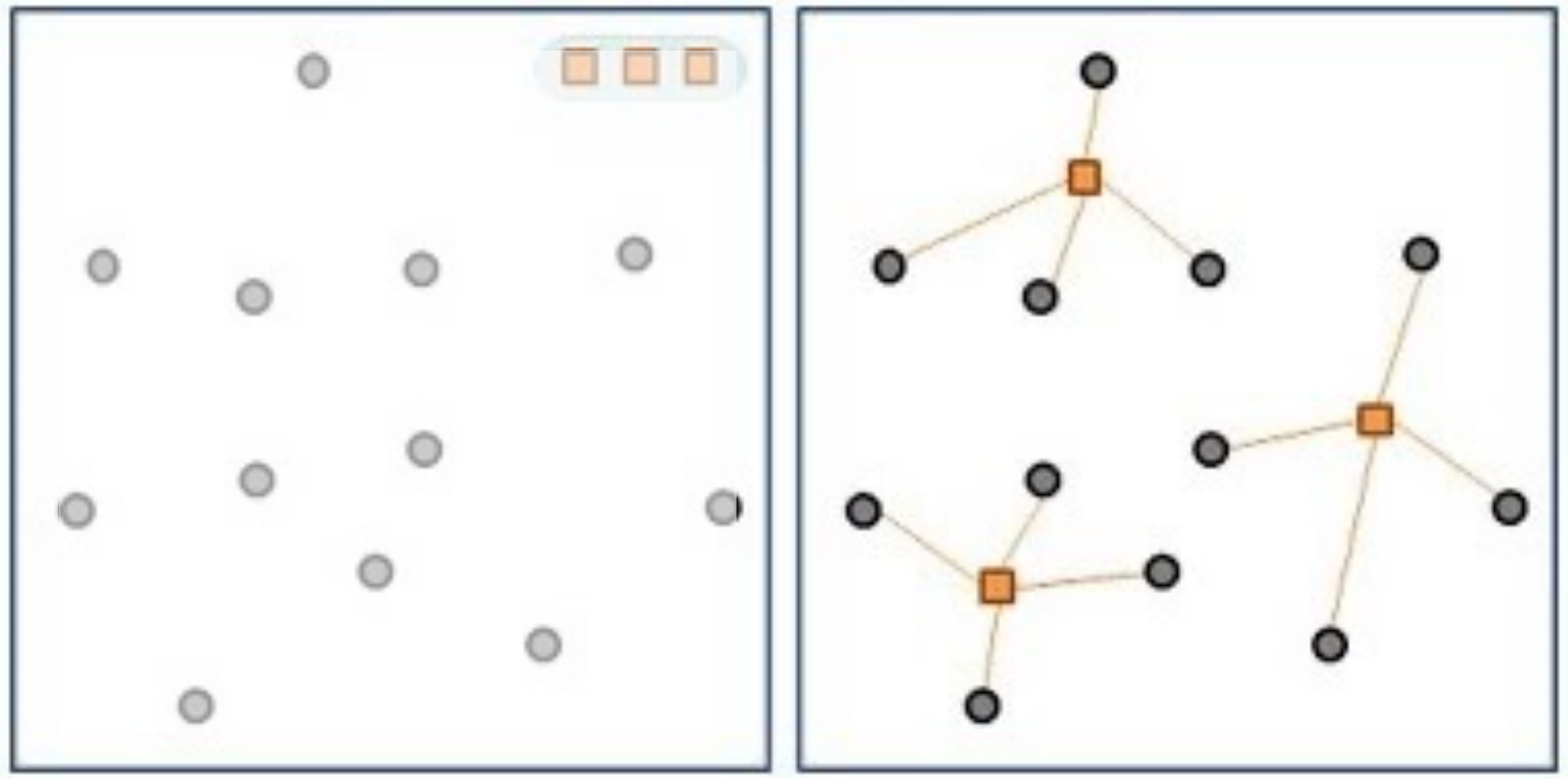

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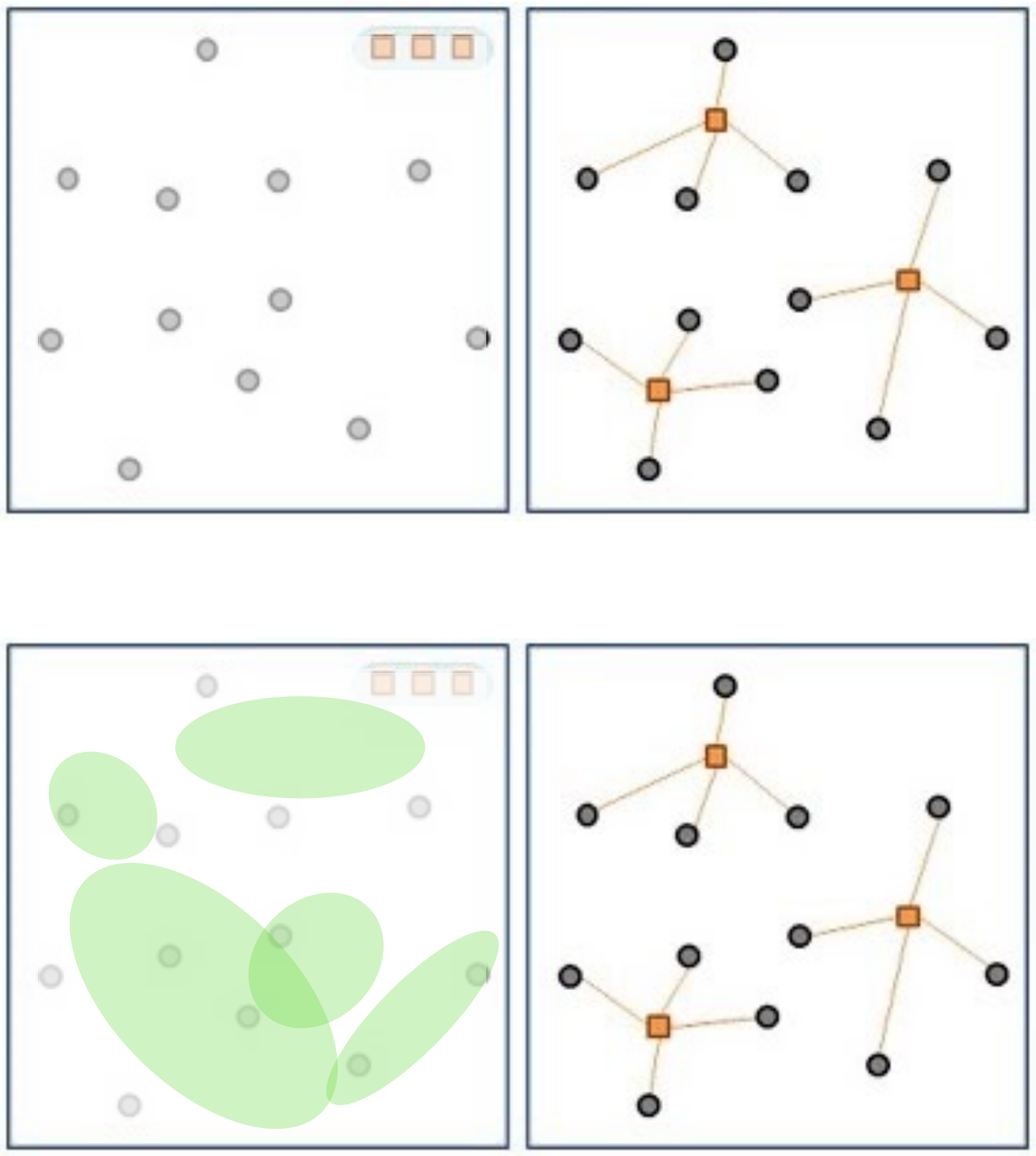

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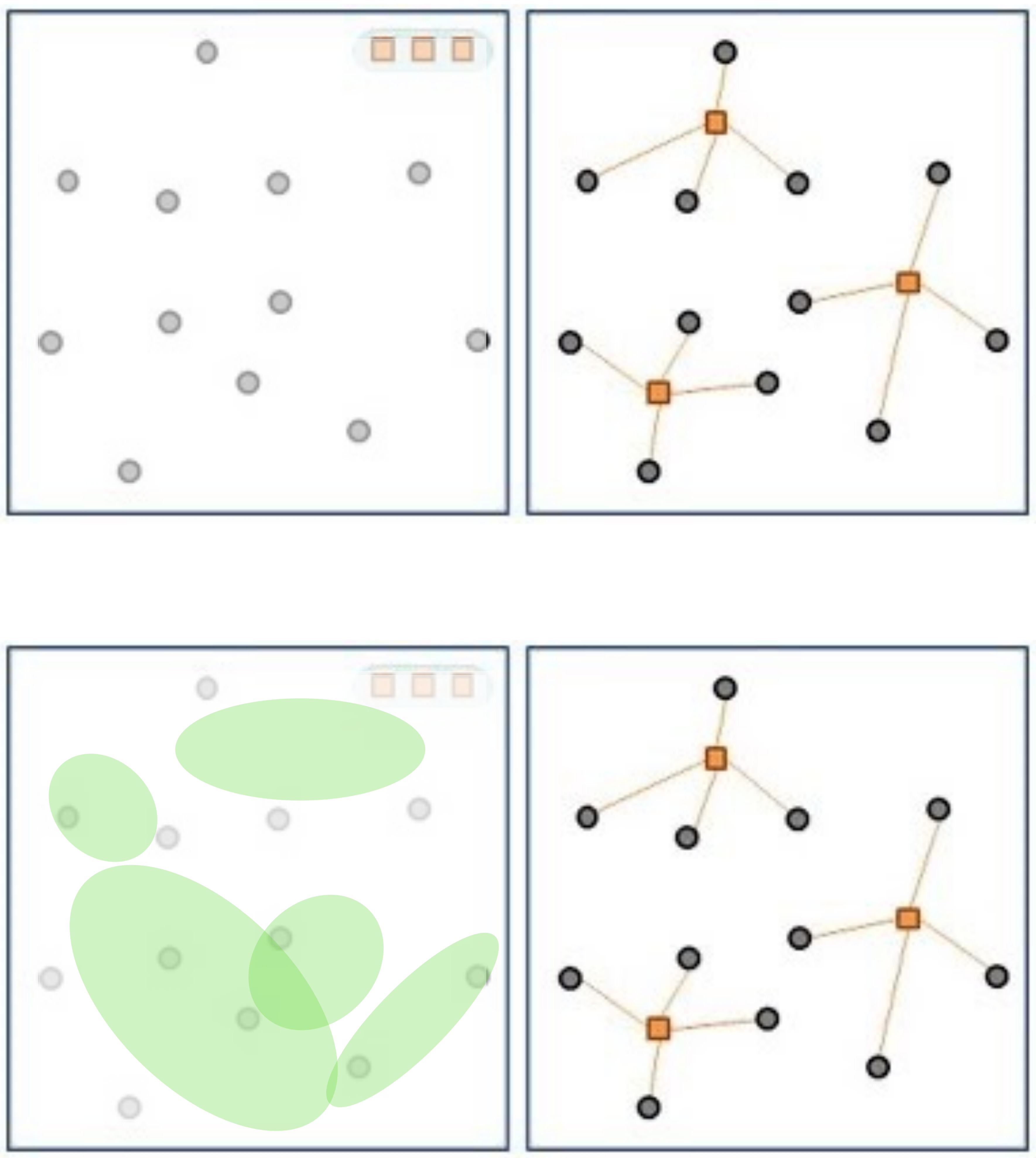

$\Rightarrow$

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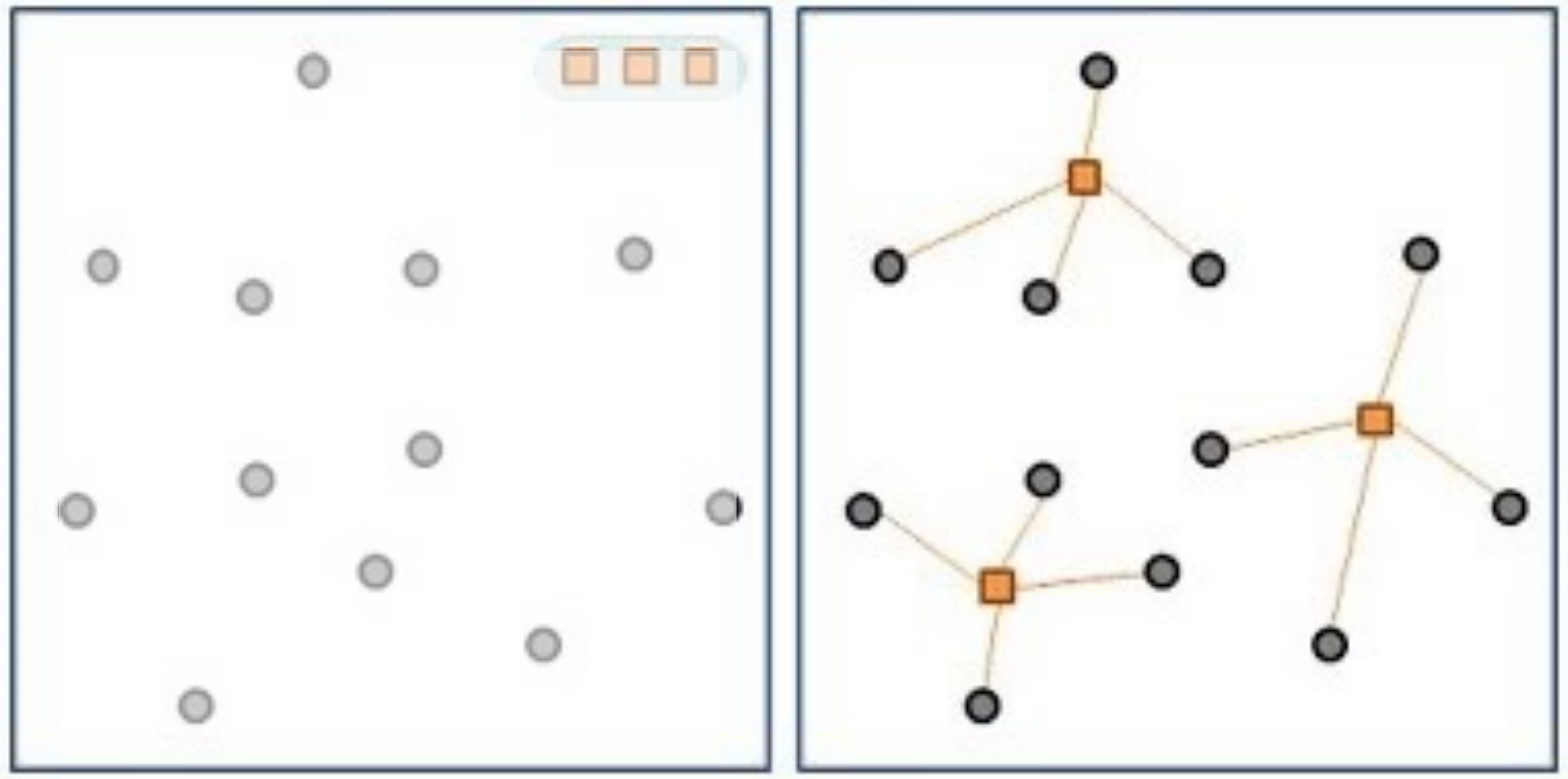
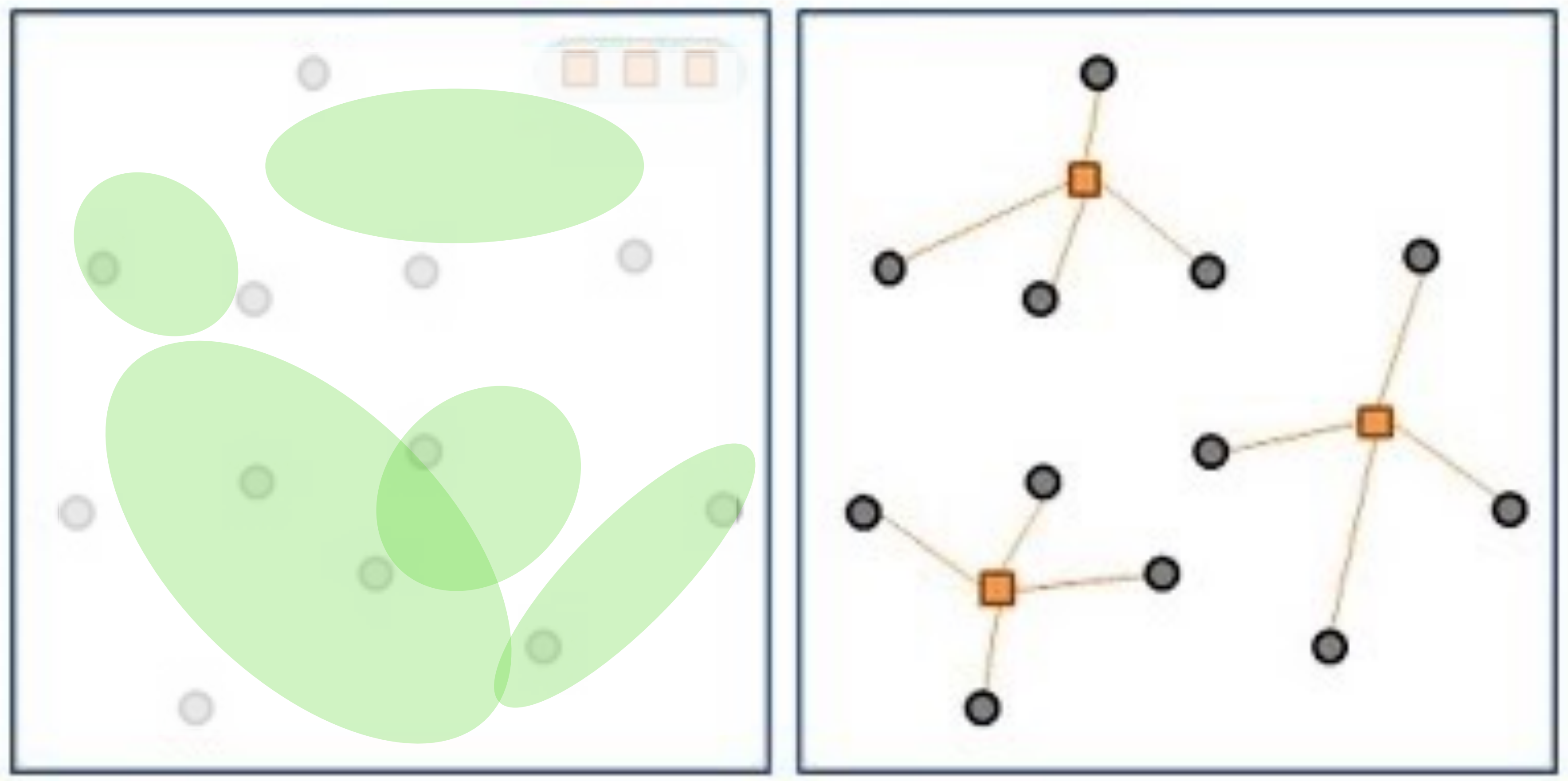


$\Rightarrow$

Prophets and Samples:



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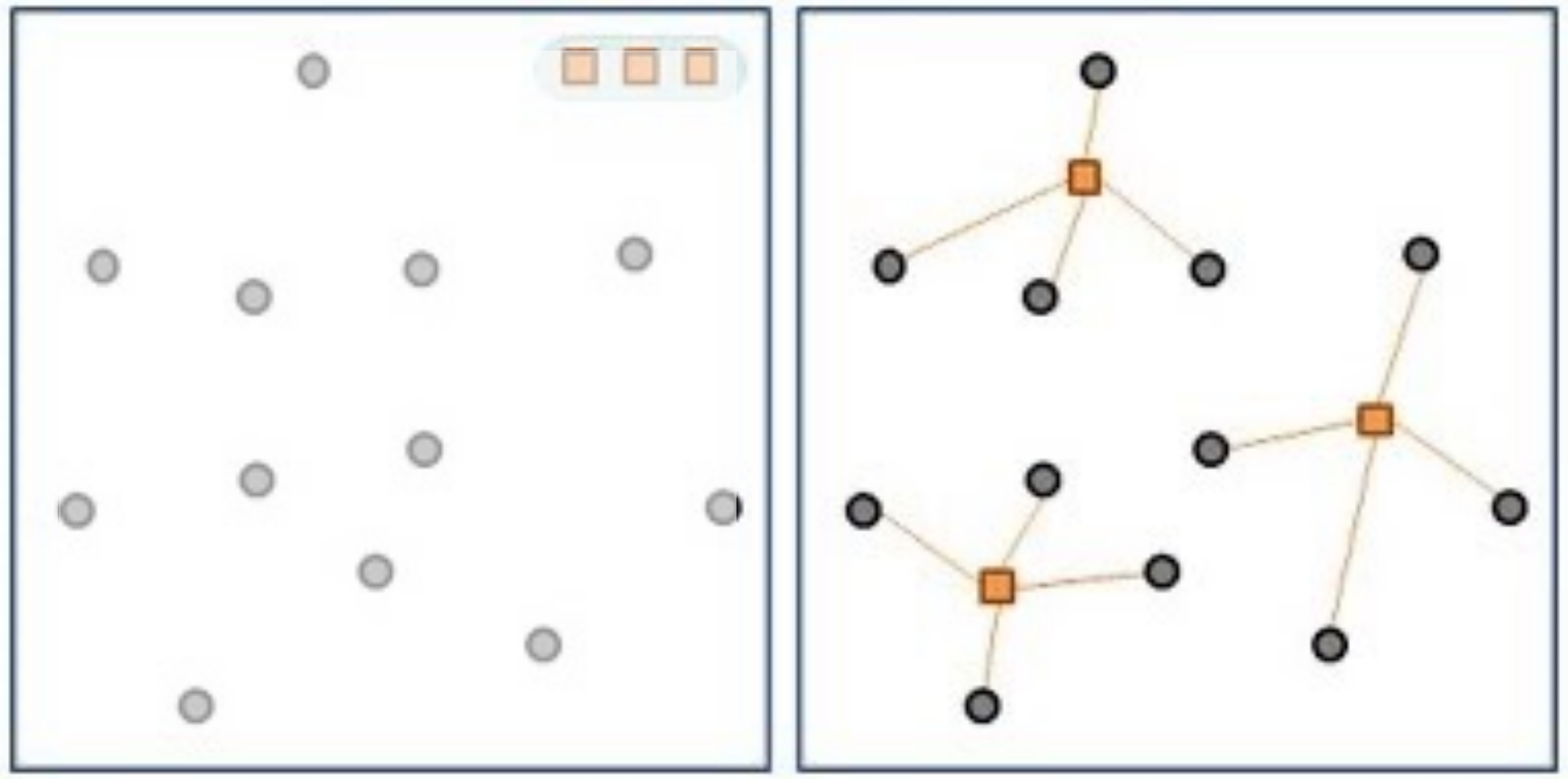
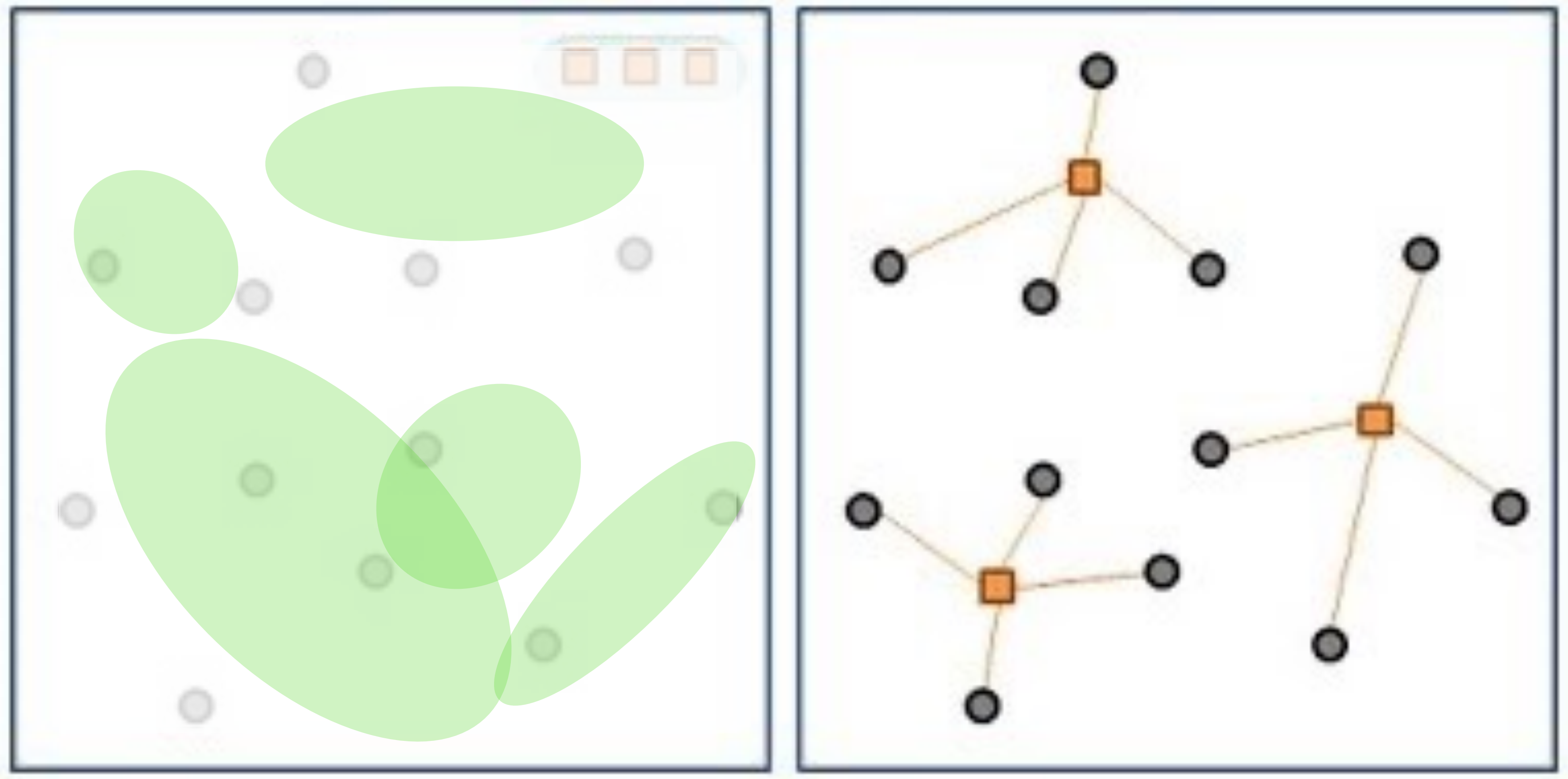


$\Rightarrow$

Prophets and Samples:

- 6-approx for single-sample prophet problem



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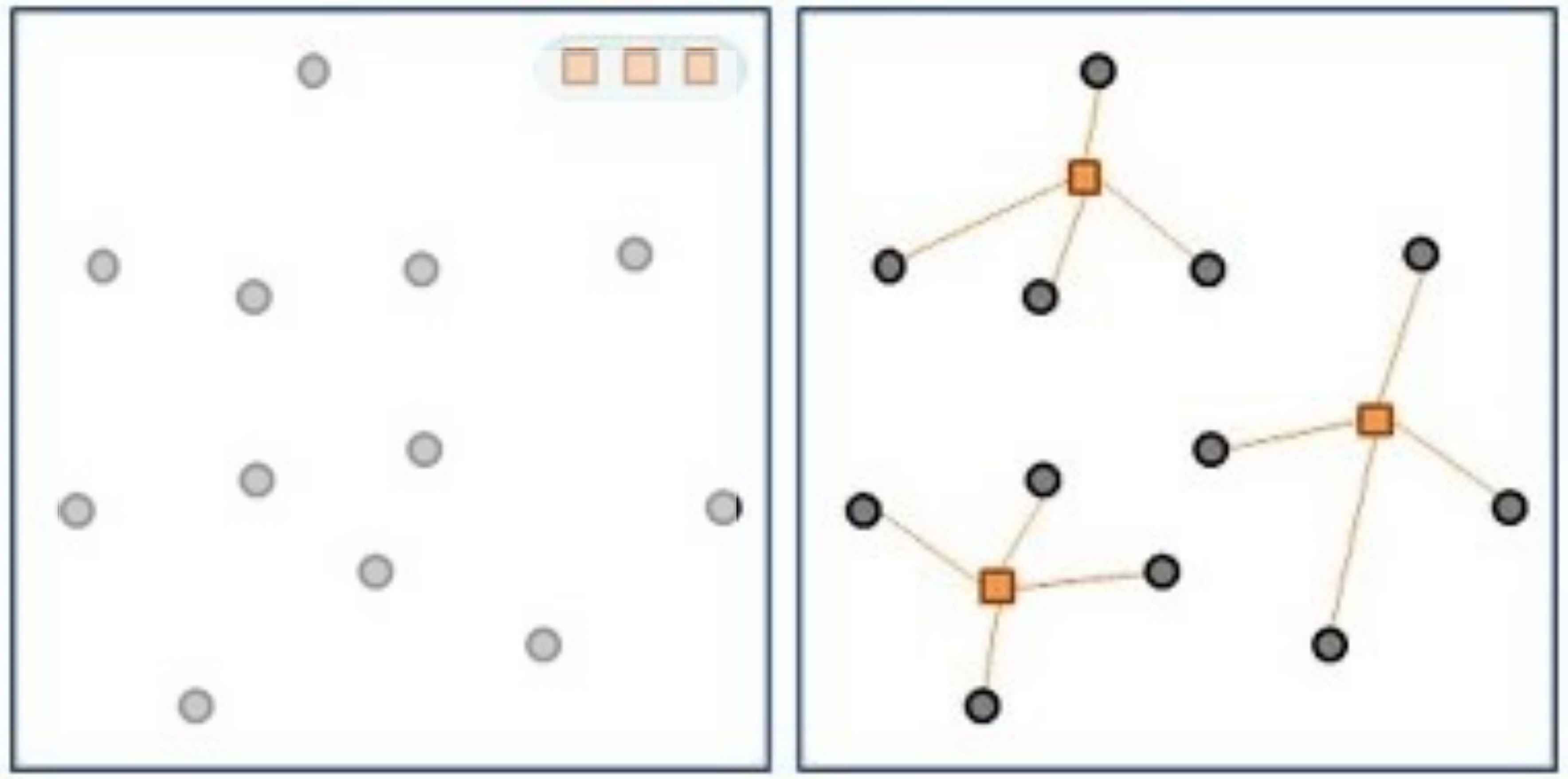
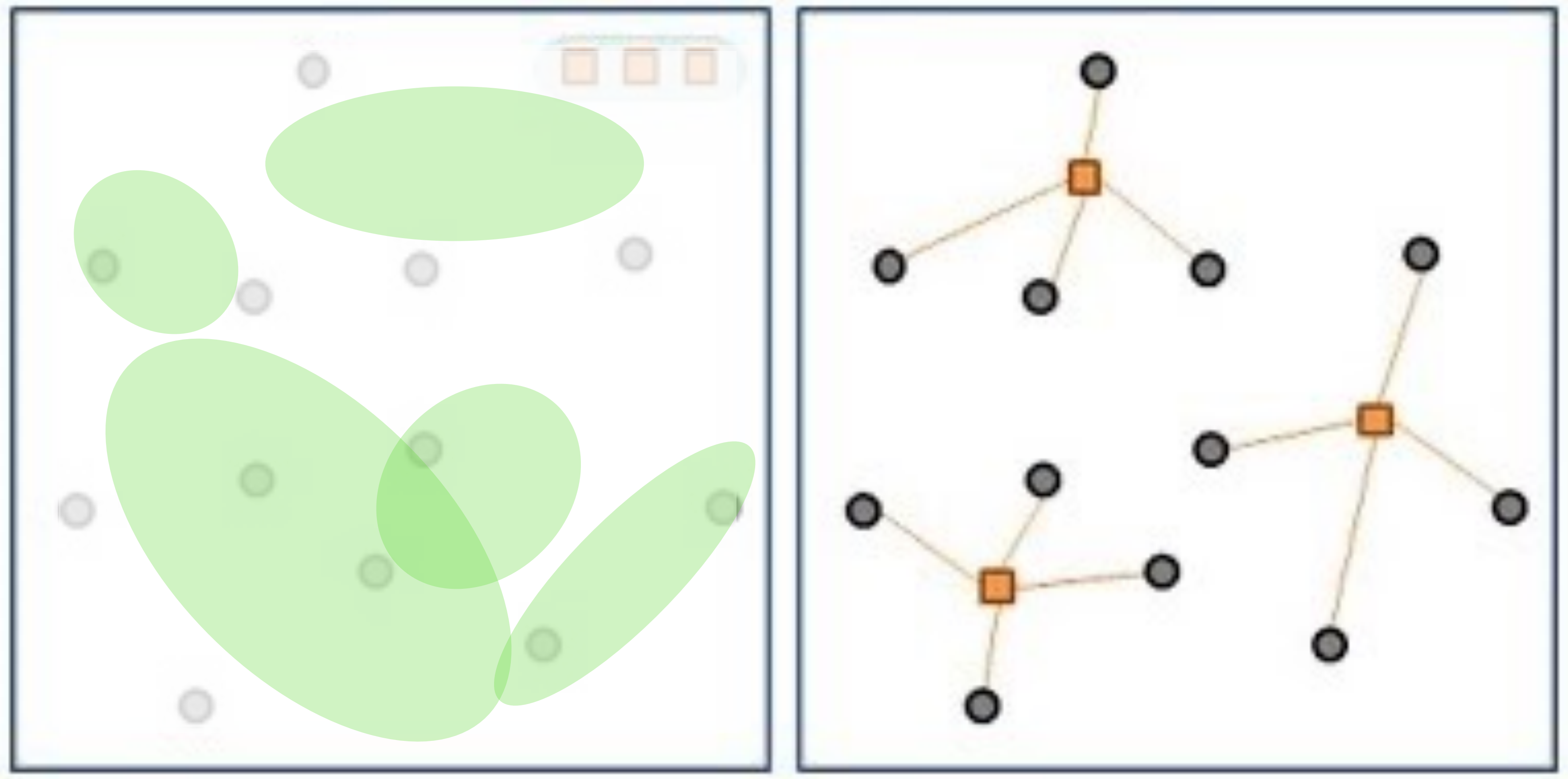


$\Rightarrow$

Prophets and Samples:

- 6-approx for single-sample prophet problem
- 6-approx for two-stage prophet problem



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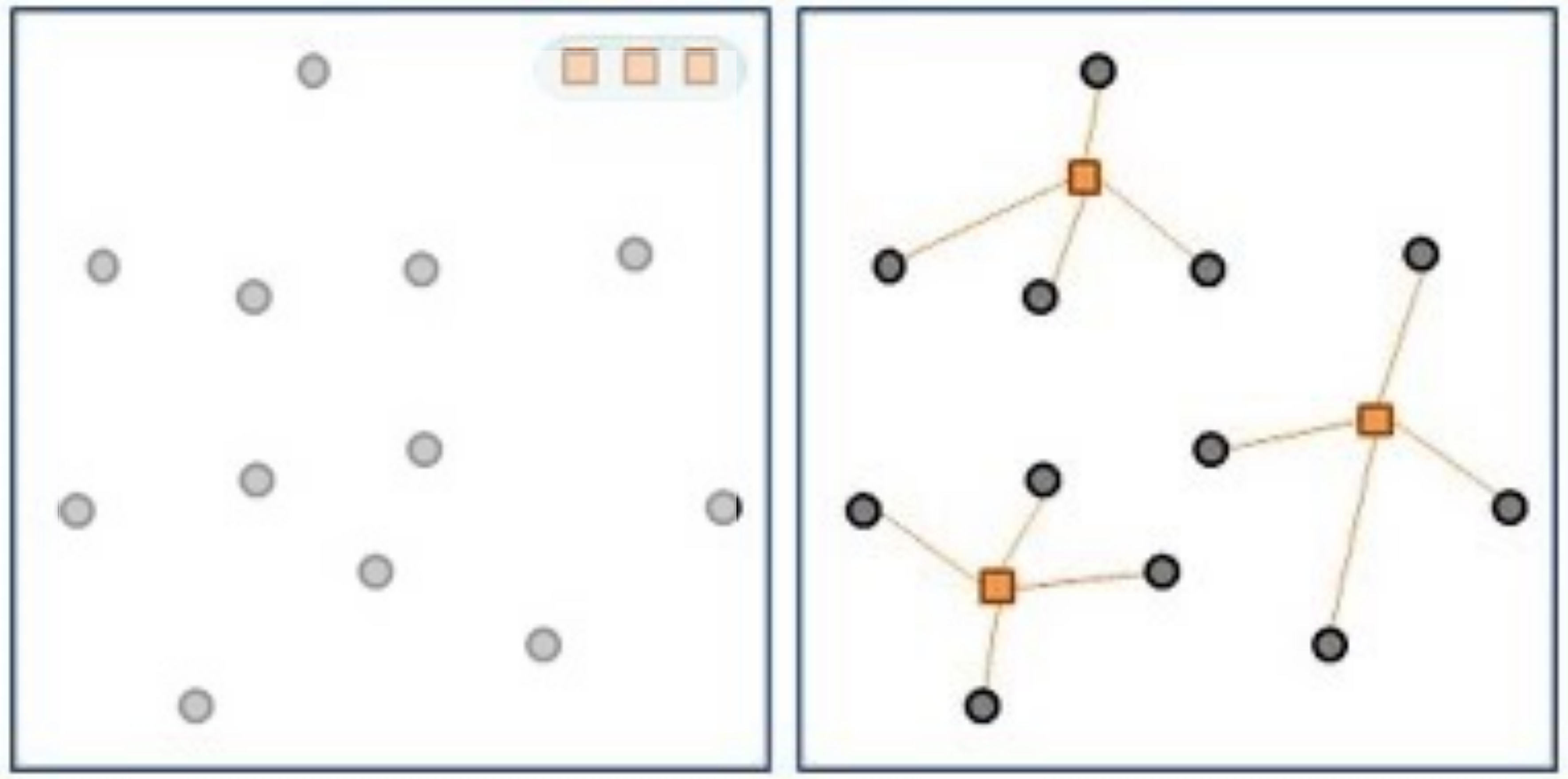
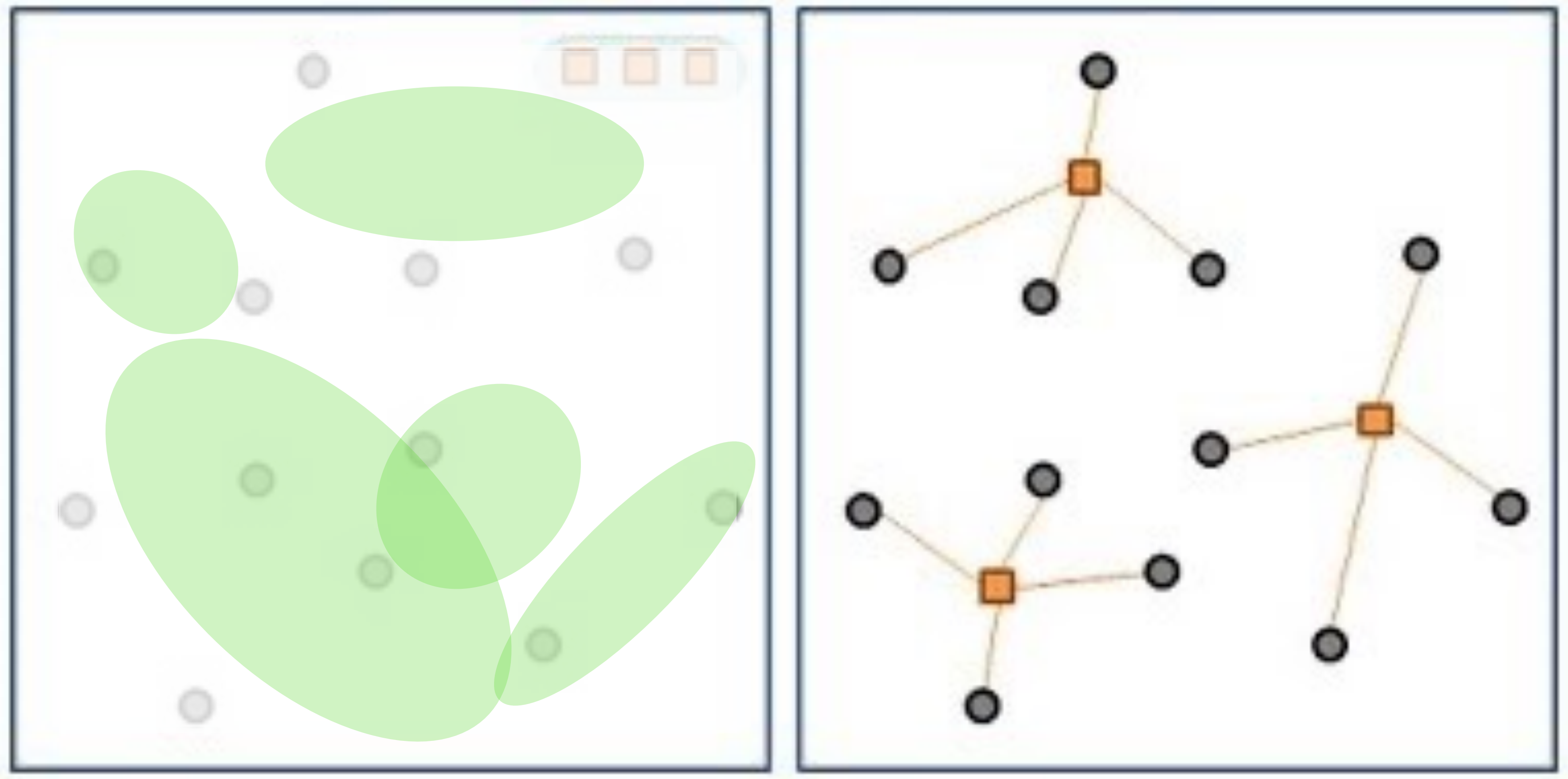


$\Rightarrow$

Prophets and Samples:

- 6-approx for single-sample prophet problem
- 6-approx for two-stage prophet problem
- $3/\alpha$-approx for with-a-sample problem



Parting Comments

Parting Comments

- Prophet algorithms are *universal* (can pre-commit to covering decisions)
- Counterpart: *packing* single-sample prophet $\rightarrow$ RO reduction [Azar Kleinberg Weinberg '14]
- Reductions work for *adaptive-order* algos, not just RO
- Dependence on α for with-a-sample results may not be tight

Thanks!

gkehne@utexas.edu